20 ott 2020

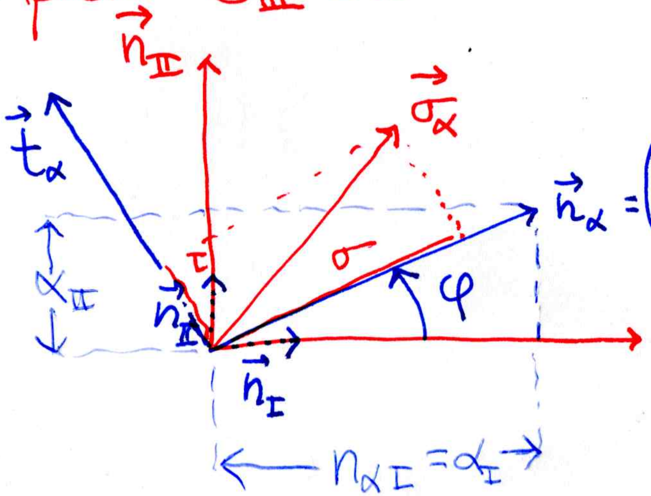
$$\vec{\sigma}_\alpha = \sigma\, \vec{n}_\alpha \longrightarrow \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{pmatrix} \xrightarrow{\text{in un riferimento privilegiato}} \begin{pmatrix} S_I & 0 & 0 \\ 0 & S_{II} & 0 \\ 0 & 0 & S_{III} \end{pmatrix}$$

Stati di sforzo piani $S_{III} = 0$

$$\sigma = \begin{pmatrix} S_I & 0 & 0 \\ 0 & S_{II} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Downarrow$$

$$\sigma = \begin{pmatrix} S_I & 0 \\ 0 & S_{II} \end{pmatrix}$$



$$\vec{n}_\alpha = \begin{pmatrix} \alpha_I \\ \alpha_{II} \end{pmatrix}$$

$$\alpha_I = |\vec{n}_\alpha| \cos\varphi = \cos\varphi$$

$$\alpha_{II} = |\vec{n}_\alpha| \sin\varphi = \sin\varphi$$

$$|\vec{n}_\alpha| = |\vec{n}_I| = |\vec{n}_{II}| = 1$$

$$\vec{\sigma}_\alpha = \begin{pmatrix} S_I & 0 \\ 0 & S_{II} \end{pmatrix} \begin{pmatrix} \alpha_I \\ \alpha_{II} \end{pmatrix} = \begin{pmatrix} S_I \alpha_I \\ S_{II} \alpha_{II} \end{pmatrix} = \begin{pmatrix} S_I \cos\varphi \\ S_{II} \sin\varphi \end{pmatrix}$$

$$\psi = \frac{1}{2}\left[a\left(\sigma_I^2+\sigma_{II}^2+\sigma_{III}^2\right)+2b\underbrace{\left(\sigma_I\sigma_{II}+\sigma_{II}\sigma_{III}+\sigma_{III}\sigma_I\right)}_{J_2}\right]$$

$$\sigma_I^2+\sigma_{II}^2+\sigma_{III}^2 = J_1^2-2J_2 \qquad a=\frac{1}{E} \qquad 2b=-\frac{\nu}{E}$$

$$\psi = \frac{1}{2E}\left(J_1^2-2J_2-2\nu J_2\right) = \frac{1}{2E}\left[J_1^2-2(1+\nu)J_2\right]$$

$$\frac{\partial\psi}{\partial\sigma_x}, \frac{\partial\psi}{\partial\sigma_y} \quad \dots \quad \frac{\partial\psi}{\partial\tau_{xy}}$$

$$J_1 = \sigma_x+\sigma_y+\sigma_z$$

$$J_2 = \sigma_x\sigma_y+\sigma_y\sigma_z+\sigma_z\sigma_x-\left(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2\right)$$

$$\psi = \frac{1}{2E}\left\{(\sigma_x+\sigma_y+\sigma_z)^2-2(1+\nu)\left[\sigma_x\sigma_y+\sigma_y\sigma_z+\sigma_z\sigma_x-\left(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2\right)\right]\right\}$$

$$= \frac{1}{2E}\left[\sigma_x^2+\sigma_y^2+\sigma_z^2+2(\sigma_x\sigma_y+\sigma_y\sigma_z+\sigma_z\sigma_x)-2(1+\nu)(\sigma_x\sigma_y+\sigma_y\sigma_z+\sigma_z\sigma_x)+\right.$$
$$\left.+2(1+\nu)\left(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2\right)\right]$$

$$= \frac{1}{2E}\left[\sigma_x^2+\sigma_y^2+\sigma_z^2-2\nu(\sigma_x\sigma_y+\sigma_y\sigma_z+\sigma_z\sigma_x)+2(1+\nu)\left(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2\right)\right]$$

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$$\varepsilon_x = \frac{\partial \psi}{\partial \sigma_x} = \frac{1}{2E}\left[2\sigma_x - 2\nu(\sigma_y + \sigma_z)\right] = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$\varepsilon_y = \frac{\partial \psi}{\partial \sigma_y} = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$\varepsilon_z = \frac{\partial \psi}{\partial \sigma_z} = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

$$\gamma_{xy} = \frac{\partial \psi}{\partial \tau_{xy}} = \frac{1}{2E}\left[4(1+\nu)\tau_{xy}\right] = \frac{2(1+\nu)}{E}\tau_{xy}$$

$\frac{1}{G}$, G modulo di taglio o di elasticità tangenziale

$$\gamma_{yz} = \frac{\tau_{yz}}{G} \quad ; \quad \gamma_{zx} = \frac{\tau_{zx}}{G}$$

$\tau_{xy} = G\,\gamma_{xy}$; $\tau_{yz} = G\,\gamma_{yz}$; $\tau_{zx} = G\,\gamma_{zx}$

$$\varepsilon_x = \frac{1}{E}\left[(1+\nu)\sigma_x - \nu(\sigma_x + \sigma_y + \sigma_z)\right]$$

$$\varepsilon_y = \frac{1}{E}\left[(1+\nu)\sigma_y - \nu(\sigma_x + \sigma_y + \sigma_z)\right]$$

$$\varepsilon_z = \frac{1}{E}\left[(1+\nu)\sigma_z - \nu(\sigma_x + \sigma_y + \sigma_z)\right]$$

$\varepsilon_x + \varepsilon_y + \varepsilon_z$

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1}{E}\left[(1+\nu)(\sigma_x+\sigma_y+\sigma_z) - 3\nu(\sigma_x+\sigma_y+\sigma_z)\right]$$

$$= \frac{1-2\nu}{E}(\sigma_x+\sigma_y+\sigma_z)$$

$$\sigma_x+\sigma_y+\sigma_z = \frac{E}{1-2\nu}(\varepsilon_x+\varepsilon_y+\varepsilon_z) \qquad 1-2\nu \neq 0$$

$$\nu \neq \frac{1}{2}$$

$$\varepsilon_x = \frac{1}{E}\left[(1+\nu)\sigma_x - \frac{\nu E}{1-2\nu}(\varepsilon_x+\varepsilon_y+\varepsilon_z)\right]$$

$$\frac{1+\nu}{E}\sigma_x = \varepsilon_x + \frac{\nu}{1-2\nu}(\varepsilon_x+\varepsilon_y+\varepsilon_z)$$

$$\sigma_x = \frac{E}{1+\nu}\left[\varepsilon_x + \frac{\nu}{1-2\nu}(\varepsilon_x+\varepsilon_y+\varepsilon_z)\right]$$

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_x + \nu(\varepsilon_y+\varepsilon_z)\right] \qquad \nu \neq -1$$

$$\sigma_y = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\,\varepsilon_y + \nu\,(\varepsilon_x + \varepsilon_z)\right]$$

$$\sigma_z = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\,\varepsilon_z + \nu\,(\varepsilon_x + \varepsilon_y)\right]$$