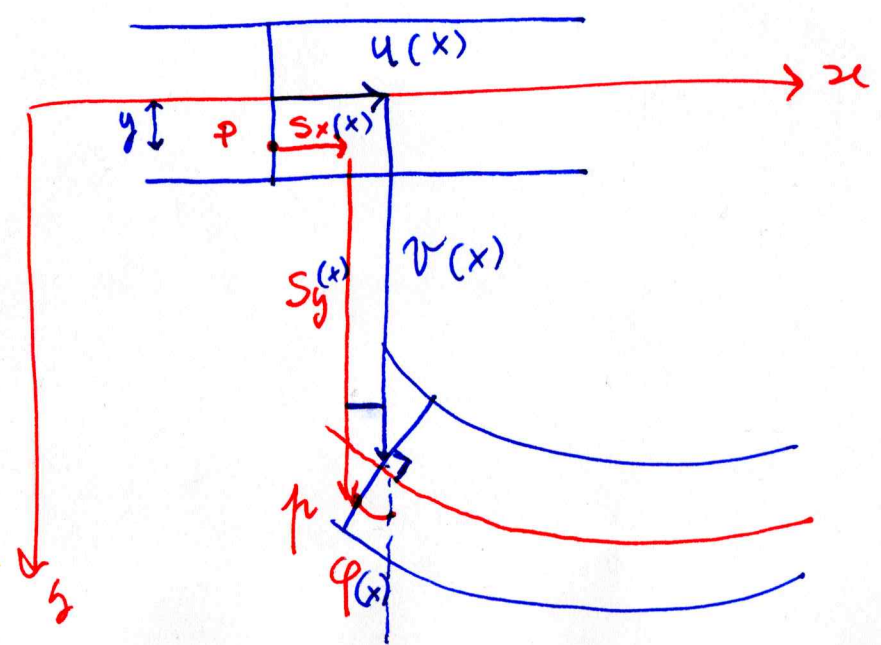


$\sin \varphi \approx \operatorname{tg} \varphi \approx \varphi$   
 $\cos \varphi \approx 1$



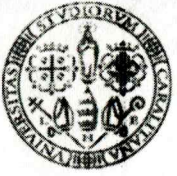
$S_x(x, y) = S_x(x) = u(x) - y \sin \varphi$

$S_x = u(x) - y \varphi(x)$

$S_y = v(x)$

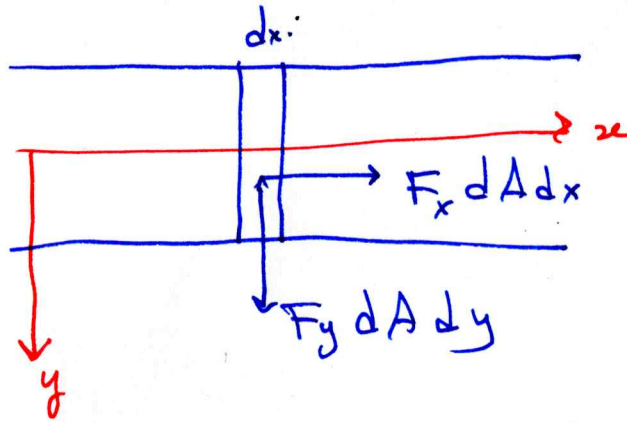
$\varphi(x) = \frac{dv(x)}{dx}$

$\begin{cases} S_x = u(x) - y \frac{dv(x)}{dx} \\ S_y = v(x) \end{cases}$



PKV

$$L_e = L_i$$



$$dV = dA dx$$

$$L_e = \int_{\Delta V} F_j \delta \hat{s}_j dV + \int_{\Delta S} f_j \delta \hat{s}_j dS$$

$$\frac{dL_e}{dx} = \int_A F_x \delta \hat{s}_x dA + \int_A F_y \delta \hat{s}_y dA$$

$$L_i = \int_{\Delta V} \sigma_{ij} \delta \hat{\epsilon}_{ij} dV$$

$$S_x = u(x) - y \frac{dv(x)}{dx}$$

$$S_y = v(x)$$

$$\frac{dL_e}{dx} = \int_A F_x \left( \delta u(x) - y \delta \frac{dv(x)}{dx} \right) dA + \int_A F_y \delta v(x) dA$$

$$v'(x) = \frac{dv(x)}{dx}$$

$$= \underbrace{\delta u \int_A F_x dA}_{n(x)} - \underbrace{\delta v'} \int_A y F_x dA}_{m(x)} + \underbrace{\delta v \int_A F_y dA}_{p(x)}$$

$$= \delta u n(x) + \delta v p(x)$$

→  $m(x) = 0$  se  $F_x$  è simmetrico rispetto a  $x$



$$\frac{dL_i}{dx} = \int_A \sigma_{ij} \delta \hat{\epsilon}_{ij} dA = \int_A (\sigma_x \delta \hat{\epsilon}_x + \sigma_y \delta \hat{\epsilon}_y + \tau_{xy} \delta \hat{\gamma}_{xy}) dA = \int_A \sigma_x \delta \hat{\epsilon}_x dA$$

$\epsilon_y = 0 \quad \gamma_{xy} = 0$

$$\epsilon_x = \eta(x) + y\chi(x)$$

$$\frac{dL_i}{dx} = \int_A \sigma_x \delta \hat{\eta}(x) dA + \int_A y \sigma_x \delta \hat{\chi}(x) dA =$$

$$= \underbrace{\delta \hat{\eta} \int_A \sigma_x dA}_{N(x)} + \underbrace{\delta \hat{\chi} \int_A y \sigma_x dA}_{M(x)}$$

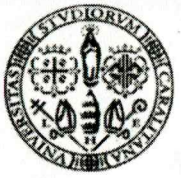
$$\eta(x) = \frac{du}{dx} = u'$$

$$\chi(x) = -\frac{d^2 v}{dx^2} = -v''$$

$$L_i = L_e$$

$$\int_0^L \{ N(x) \delta u' - M(x) \delta v'' \} dx = \int_0^L \{ n(x) \delta u + p(x) \delta v \} dx +$$

$$+ H_0 \delta u(0) + V_0 \delta v(0) + W_0 \delta v'(0) + H_L \delta u(L) + V_L \delta v(L) + W_L \delta v'(L)$$



$$D(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$$

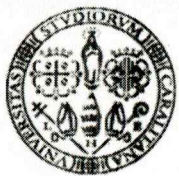
$$\int_a^b f'(x)g(x) dx = \int_a^b D(f(x)g(x)) dx - \int_a^b f(x)g'(x) dx$$

$$\int_a^b f'(x)g(x) dx = f(x)g(x) \Big|_a^b - \int_a^b f(x)g'(x) dx$$

$$\int_0^L N(x) \delta u'(x) dx = N(x) \delta u(x) \Big|_0^L - \int_0^L N'(x) \delta u(x) dx$$

$$\int_0^L -M(x) \delta v''(x) dx = -M(x) \delta v'(x) \Big|_0^L + \int_0^L M'(x) \delta v'(x) dx$$

$$= -M(x) \delta v'(x) \Big|_0^L + M'(x) \delta v(x) \Big|_0^L - \int_0^L M''(x) \delta v(x) dx$$



$$N(L) \delta u(L) - N(0) \delta u(0) - \int_0^L N'(x) \delta u(x) dx - M(L) \delta v'(L) + M(0) \delta v'(0) +$$

$$+ M'(L) \delta v(L) - M'(0) \delta v(0) - \int_0^L M''(x) \delta v(x) dx$$


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$$\int_0^L \left\{ - [N'(x) + n(x)] \delta u(x) - [M''(x) + p(x)] \delta v(x) \right\} dx =$$

$$= (H_0 + N(0)) \delta u(0) + (H_L - N(L)) \delta u(L) + (V_0 + M'(0)) \delta v(0) + (V_L - M'(L)) \delta v(L) +$$

$$+ (-M(0) + W_0) \delta v'(0) + (M(L) + W_L) \delta v'(L)$$

$$N'(x) + n(x) = 0$$

$$M''(x) + p(x) = 0$$

$$\delta u(0) = 0$$

$$\delta u(L) = 0$$

$$\delta v(0) = 0$$

$$\delta v(L) = 0$$

$$\delta v'(0) = 0$$

$$\delta v'(L) = 0$$

oppure

$$N(0) = -H_0$$

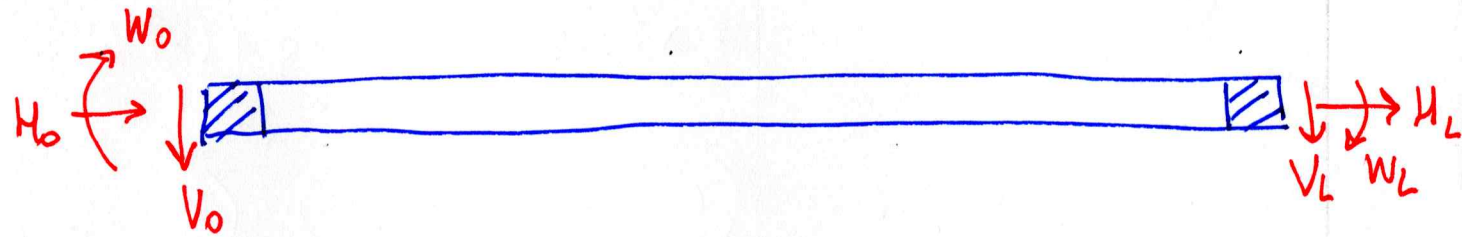
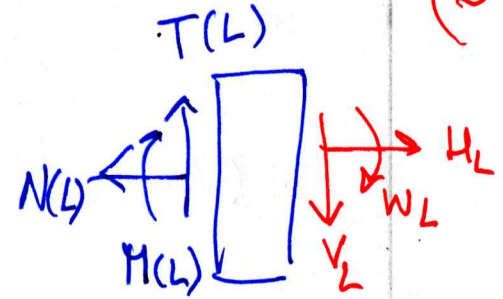
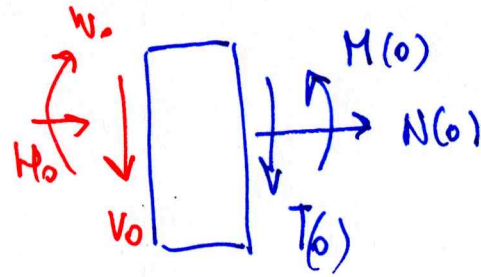
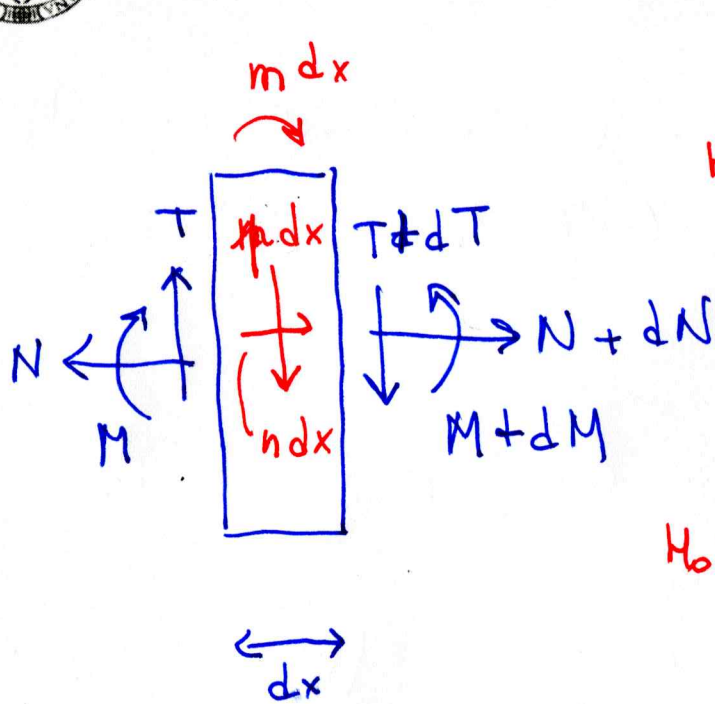
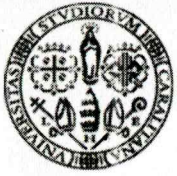
$$N(L) = H_L$$

$$M'(0) = -V_0$$

$$M'(L) = V_L$$

$$M(0) = W_0$$

$$M(L) = -W_L$$



$$N'(x) + n(x) = 0$$

$$M''(x) + p(x) = 0$$

$$T = M'(x)$$

$$N(x) = EA \eta(x)$$

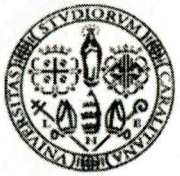
$$= EA u'(x)$$

$$M(x) = EJ \chi(x)$$

$$= -EJ v''(x)$$

$$\eta(x) = \frac{du(x)}{dx} = u'(x)$$

$$\chi(x) = -\frac{d^2v(x)}{dx^2} = -v''(x)$$



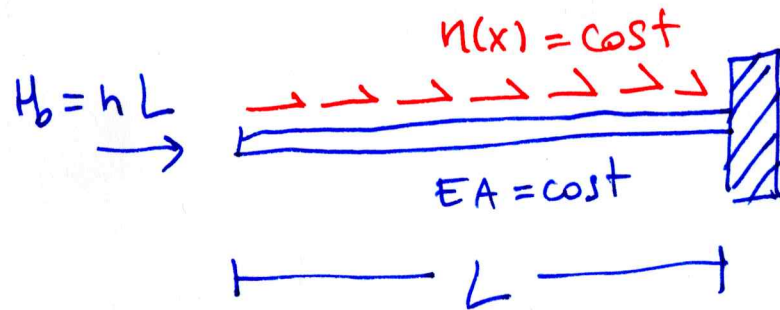
$$\left[EA u'(x)\right]' + n(x) = 0$$

$$\left[-EJ v''(x)\right]'' + p(x) = 0$$

$$u''(x) = -\frac{n(x)}{EA}$$

$$v''''(x) = \frac{p(x)}{EJ}$$

### ESERCIZIO 1



### ESERCIZIO 2

