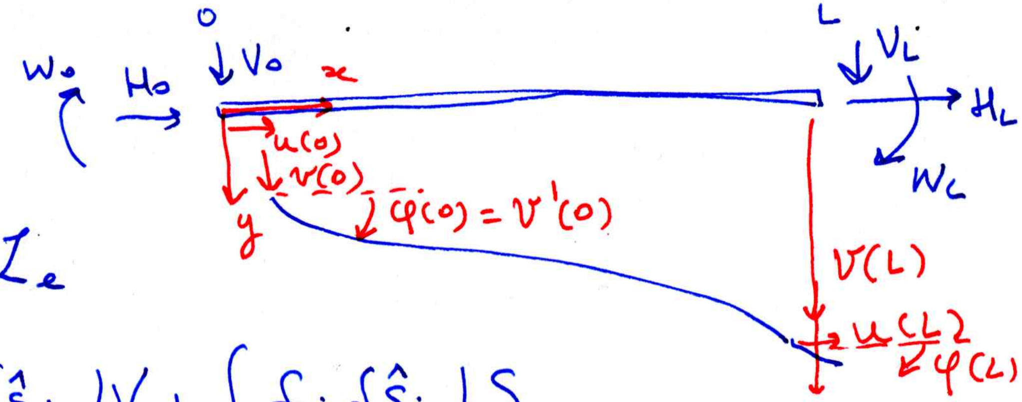
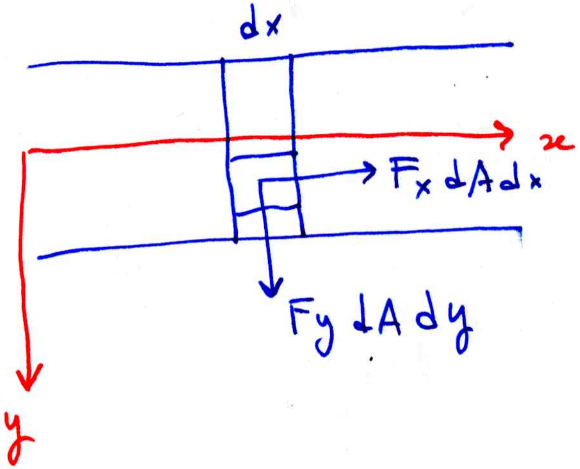
Resumé delle puntate precedenti





$$\mathcal{L}_i = \mathcal{L}_e$$

$$\mathcal{L}_e = \int_{\Delta V} F_j\, \delta \hat{s}_j\, dV + \int_{\Delta S} f_j\, \delta \hat{s}_j\, dS$$

$$\mathcal{L}_i = \int_{\Delta V} \sigma_{ij}\, \delta \hat{\varepsilon}_{ij}\, dV \qquad \delta \hat{u} \quad \delta \hat{v}$$

$$\int_0^L \left\{ N(x)\, \delta \underline{u'(x)} - M(x)\, \delta \underline{v''(x)} \right\} dx = \int_0^L \left\{ n(x)\, \delta u(x) + p(x)\, \delta v(x) \right\} dx +$$

$$+ H_0\, \delta u(0) + V_0\, \delta v(0) + W_0\, \delta v'(0) +$$

$$+ H_L\, \delta u(L) + V_L\, \delta v(L) + W_L\, \delta v'(L)$$

$$\int_{x_1}^{x_2} f'(x)\, g(x)\, dx = f(x)\, g(x) \Big|_{x_1}^{x_2} - \int f(x)\, g'(x)\, dx$$

$$D\left[f(x)\, g(x) \right] = f'(x)\, g(x) + f(x)\, g'(x)$$

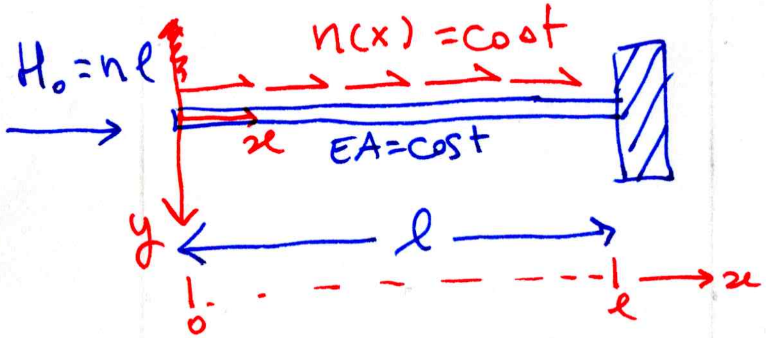
$$\int_0^L \left\{ -\left[N'(x)+n(x)\right]\delta u(x) - \left[M''(x)+p(x)\right]\delta v(x) \right\} dx =$$

$$= \left(H_0 + N(0)\right)\delta u(0) + \left(H_L - N(L)\right)\delta u(L) + \left(V_0 + M'(0)\right)\delta v(0) + \left(V_L - M'(L)\right)\delta v(L) +$$

$$+ \left(-M(0) + W_0\right)\delta v'(0) + \left(M(L) + W_L\right)\delta v'(L)$$

$$\begin{cases} N'(x) + n(x) = 0 \\ M''(x) + p(x) = 0 \end{cases} \qquad \delta u(0) = 0 \qquad u(0) = \bar{u}$$

ESERCIZIO 1



$$EA\,u''(x) = -n \qquad N(0) = -H_0 = EA\,u'(0)$$

$$u(\ell) = 0$$

$n = \text{cost}$!

$$u''(x) = -\frac{n}{EA} \;; \qquad u'(x) - \frac{nx}{EA} + B$$

$$u(x) = -\frac{1}{2}\frac{n x^2}{EA} + Bx + C$$

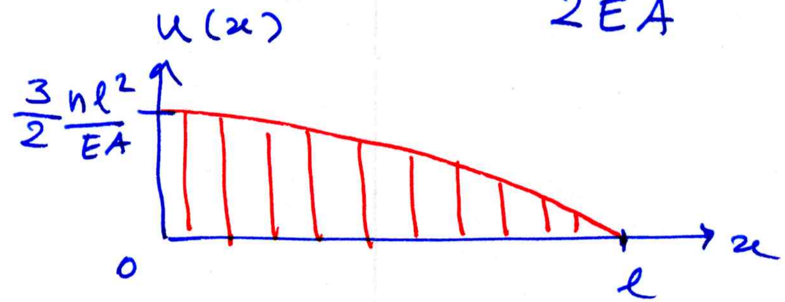
$$u'(0) = -\frac{H_0}{EA} \Rightarrow -\frac{n \cdot 0}{EA} + B = -\frac{H_0}{EA} \Rightarrow B = -\frac{H_0}{EA}$$

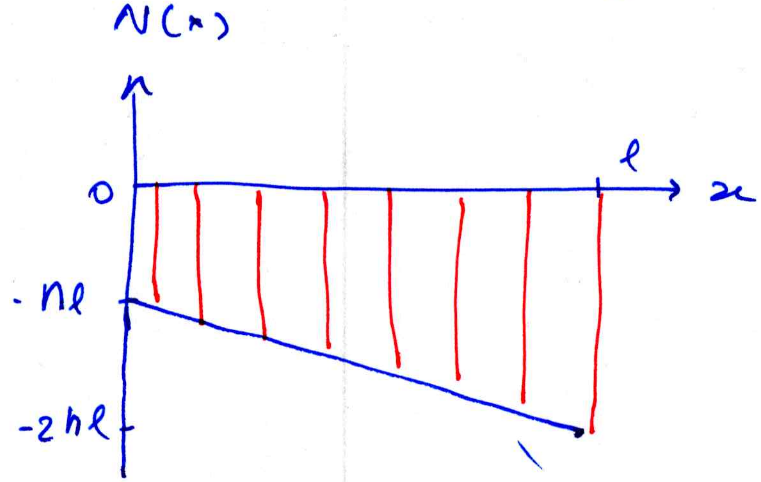
$$u(\ell) = 0 \Rightarrow -\frac{1}{2}\frac{n\ell^2}{EA} + B\ell + C = 0 \qquad H_0 = n\ell$$

$$-\frac{1}{2}\frac{n\ell^2}{EA} - \frac{n\ell^2}{EA} + C = 0 \Rightarrow C = \frac{3n\ell^2}{2EA}$$

$$u(x) = \frac{1}{2}\frac{n}{EA}\left(3\ell^2 - 2\ell x - x^2\right)$$

$$N(x) = -n(\ell + x)$$





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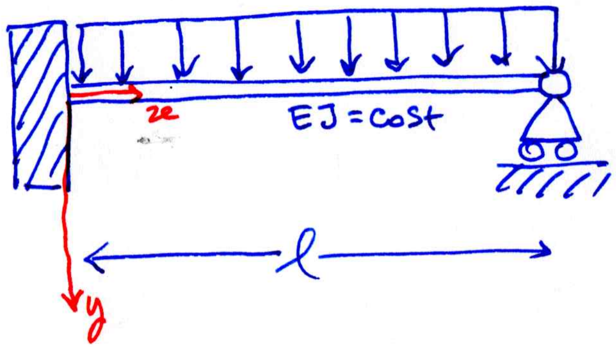
Esercizio 2



$\downarrow p(x) = p_0 \qquad v^{IV}(x) = \frac{p_0}{EJ}$

① $v(0) = 0$ ② $v'(0) = 0$ ③ $v(l) = 0$

④ $M(l) = 0 \Rightarrow -EJ\, v''(l) = 0 \Rightarrow v''(l) = 0$

integro l'equazione differenziale

$v'''(x) = \frac{p_0 x}{EJ} + C_1 \quad ; \quad v''(x) = \frac{1}{2}\frac{p_0}{EJ}x^2 + C_1 x + C_2$

$v'(x) = \frac{1}{6}\frac{p_0}{EJ}x^3 + \frac{1}{2}C_1 x^2 + C_2 x + \cancel{C_3} \quad ; \quad v(x) = \frac{1}{24}\frac{p_0}{EJ}x^4 + \frac{1}{6}C_1 x^3 + \frac{1}{2}C_2 x^2 + \cancel{C_3} x + \cancel{C_4}$

① $v(0) = C_4 = 0$ ② $v'(0) = C_3 = 0$

③ $v(l) = 0 \Rightarrow v(l) = \boxed{\frac{p_0 l^4}{24EJ} + \frac{C_1 l^3}{6} + \frac{C_2 l^2}{2} = 0}$

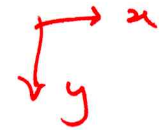
④ $v''(l) = 0 \Rightarrow v''(l) = \boxed{\frac{p_0 l^2}{2EJ} + C_1 l + C_2 = 0}$

$C_1 = -\frac{5}{8}\frac{p_0 L}{EJ}$

$C_2 = \frac{1}{8}\frac{p_0 L^2}{EJ}$

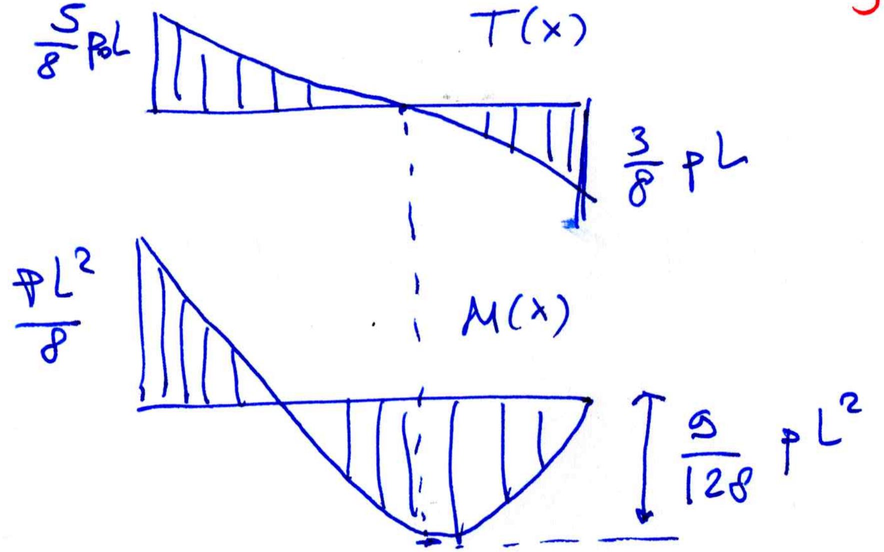
$C_3 = C_4 = 0$

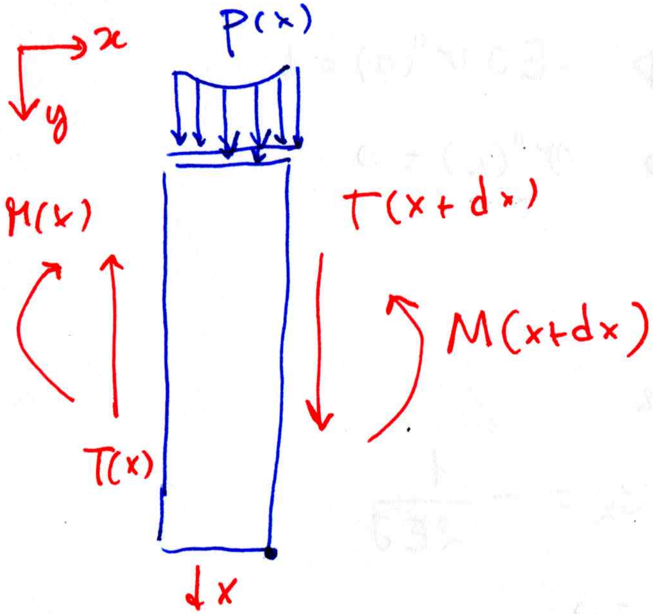
$$M(x) = -EJ v''(x) \quad e \quad T(x) = -EJ v'''(x)$$



$$v''(x) = \frac{p_0}{2EJ} x^2 - \frac{5}{8} \frac{p_0 l}{EJ} x + \frac{p_0 l^2}{8EJ}$$

$$v'''(x) = \frac{p_0 x}{EJ} - \frac{5}{8} \frac{p_0 l}{EJ}$$





$$T(x+dx) = T(x) + \frac{dT(x)}{dx}\,dx$$

$$M(x+dx) = M(x) + \frac{dM(x)}{dx}\,dx$$

EQUILIBRIO

$R_y = 0$

$M_z = 0$

$R_y = 0$

$$-T(x) + T(x+dx) + p(x)\,dx = 0$$

$$-T(x) + T(x) + \frac{dT(x)}{dx}\,dx + p(x)\,dx = 0$$

$$\boxed{\frac{dT(x)}{dx} = -p(x)}$$

$M_z = 0$

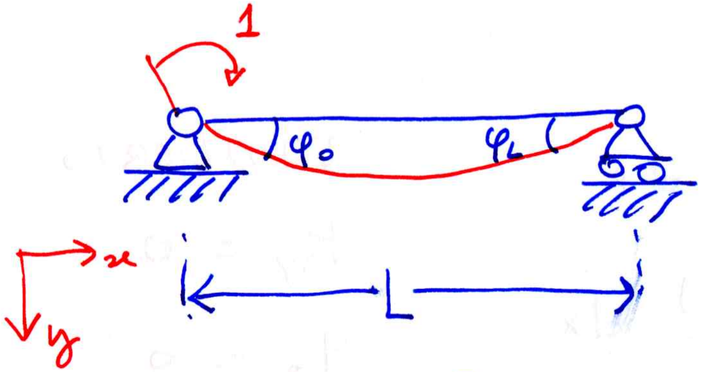
$$M(x) - M(x+dx) + T(x)\,dx - p(x)\,\frac{dx \cdot dx}{2} = 0$$

$\sim dx^2 << dx$

$$M(x) - M(x) - \frac{dM(x)}{dx}\,dx + T(x)\,dx = 0$$

$$\boxed{\frac{dM(x)}{dx} = T(x)}$$

ESERCIZIO 3



$$v^{IV}(x)=0$$

$$v(x)=C_1x^3+C_2x^2+C_3x+\cancel{C_4}$$

① $v(0)=0$ ② $M(0)=1 \Rightarrow -EJv''(0)=1$

③ $v(L)=0$ ④ $M(L)=0 \Rightarrow v''(L)=0$

dalla ① $v(0)=C_4=0$

$v'(x)=3C_1x^2+2C_2x+C_3$; $v''(x)=6C_1x+2C_2$

dalla ② $v''(0)=2C_2 \Rightarrow v''(0)=-\dfrac{1}{EJ} \Rightarrow C_2=-\dfrac{1}{2EJ}$

dalla ③ $v(L)=0 \Rightarrow v(L)=C_1L^3-\dfrac{L^2}{2EJ}+C_3L=0$

dalla ④ $v''(L)=0 \Rightarrow v''(L)=6C_1L-\dfrac{1}{EJ}=0 \Rightarrow C_1=\dfrac{1}{6EJL}$

$$C_1L^3-\frac{L^2}{2EJ}+C_3L=\frac{L^2}{6EJ}-\frac{L^2}{2EJ}+C_3L=0$$

$$C_3=\frac{L}{2EJ}-\frac{L}{6EJ}=\frac{L}{3EJ}$$

$C_1=\dfrac{1}{6EJL}$

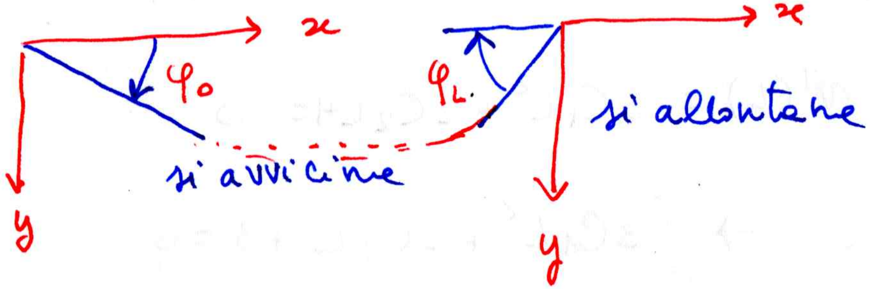
$C_2=-\dfrac{1}{2EJ}$

$C_3=\dfrac{L}{3EJ}$

$C_4=0$

$$v(x) = \frac{x^3}{6EJL} - \frac{x^2}{2EJ} + \frac{xL}{3EJ} \; ; \quad v'(x) = \frac{x^2}{2EJL} - \frac{x}{EJ} + \frac{L}{3EJ}$$

$$\varphi_0 = v'(0) = \frac{L}{3EJ} \; ; \quad \varphi_L = -v'(L) = -\left(\frac{L}{2EJ} - \frac{L}{EJ} + \frac{L}{3EJ}\right) = \frac{L}{6EJ}$$

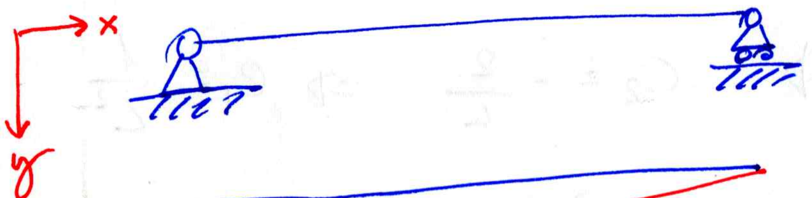


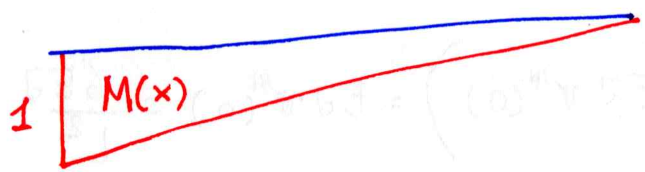
$$v''(x) = \frac{x}{EJL} - \frac{1}{EJ}$$

$$M(x) = -EJ\, v''(x) = 1 - \frac{x}{L}$$

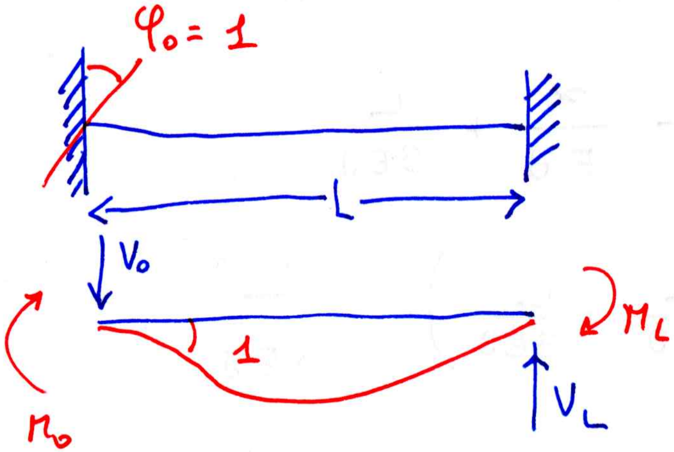
$$v'''(x) = \frac{1}{EJL}$$

$$T(x) = -EJ\, v'''(x) = -\frac{1}{L}$$





Esercizio 4



$$v^{IV}(x) = 0 \Rightarrow v(x) = C_1 x^3 + C_2 x^2 + \cancel{C_3}^{\,1} x + \cancel{C_4}$$

Condizioni al bordo

① $v(0) = 0$ ② $v'(0) = 1$ ③ $v(L) = 0$ ④ $v'(L) = 0$

$$v'(x) = 3C_1 x^2 + 2C_2 x + \cancel{C_3}^{\,1}$$

dalla ① $v(0) = C_4 = 0$ dalla ② $v'(0) = C_3 = 1$

dalla ③ $v(L) = C_1 L^3 + C_2 L^2 + L = 0$ dalla ④ $v'(L) = 3C_1 L^2 + 2C_2 L + 1 = 0$

$C_1 L^2 + C_2 L + 1 = 0$ moltiplico tutto per 3 $\rightarrow$ $3C_1 L^2 + 3C_2 L + 3 = 0$

sottraggo la ④

$$\begin{array}{r} 3C_1 L^2 + 3C_2 L + 3 = 0 \; - \\ 3C_1 L^2 + 2C_2 L + 1 = 0 \\ \hline / \quad C_2 L + 2 = 0 \end{array} \Rightarrow C_2 = -\frac{2}{L} \Rightarrow C_1 = \frac{1}{L^2}$$

$$v(x) = \frac{x^3}{L^2} - \frac{2x^2}{L} + x$$

$$v'(x) = \frac{3x^2}{L^2} - \frac{4x}{L} + 1$$

$$v''(x) = \frac{6x}{L^2} - \frac{4}{L}$$

$$v'''(x) = \frac{6}{L^2}$$

$$V_0 = -T(0) = -\left(-EJ\, v'''(0)\right) = EJ\, v'''(0) = \frac{6EJ}{L^2}$$

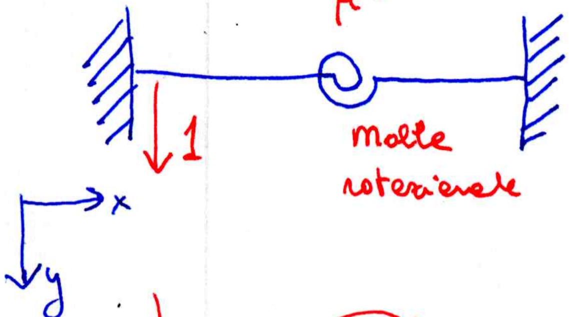
$$V_L = T(L) = -EJ\, v'''(L) = -\frac{6EJ}{L^2}$$

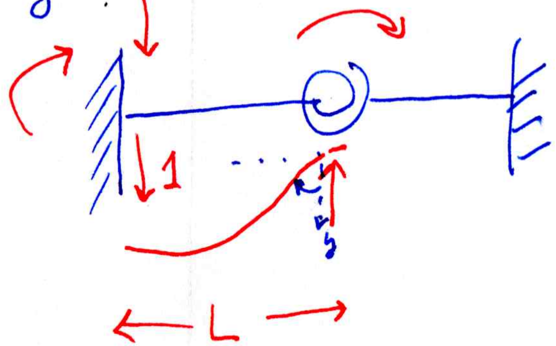
$$M_0 = M(0) = -EJ\, v''(0) = -EJ\left(-\frac{4}{L}\right) = \frac{4EJ}{L}$$

$$M_L = -M(L) = EJ\, v''(L) = \frac{2EJ}{L}$$

ESERCIZIO 5 : VINCOLO ELASTICO

μ → costante elastica della molla rotazionale





$$v^{IV}(x) = 0 \Rightarrow v(x) = C_1 x^3 + C_2 x^2 + C_3 x + C_4$$

Condizioni al bordo

① $v(0) = 1$ ② $v'(0) = 0$

③ $v(L) = 0$

$$\varphi_L = -\mu\, v'(L) \qquad M(L) = -EJ\, v''(L)$$

④ $-\mu\, v'(L) + EJ\, v''(L) = 0$