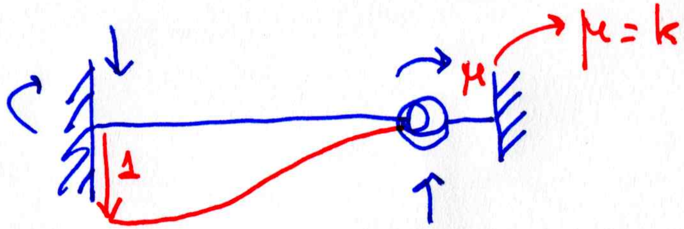
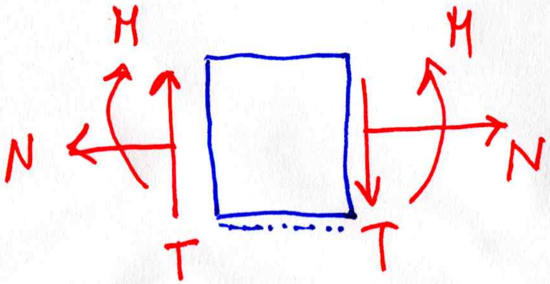
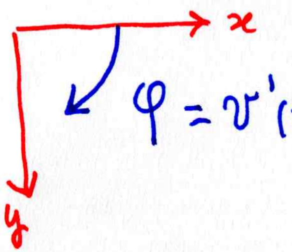
ESERCIZIO: VINCOLO ELASTICO



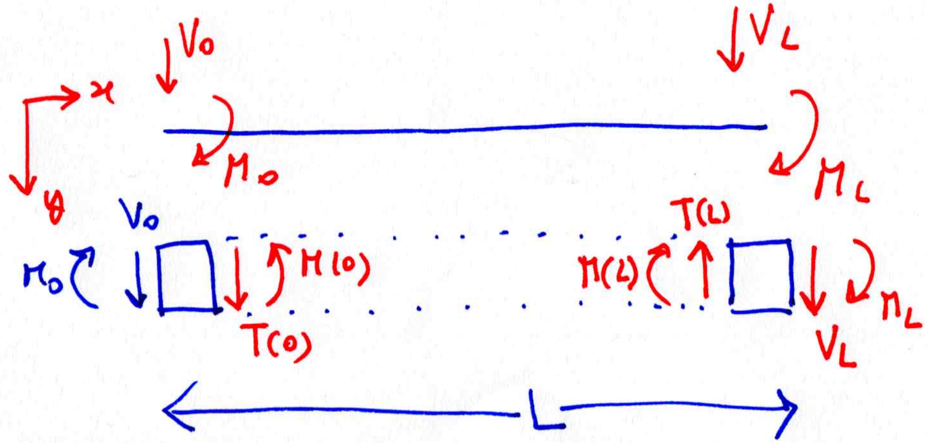
$$v^{IV}(x) = \frac{p(x)}{EJ} \Rightarrow v^{IV}(x) = 0$$

$$v(x) = c_1 x^3 + c_2 x^2 + c_3 x + c_4 = A x^3 + B x^2 + C x + D$$





$$\varphi = v'(x)$$

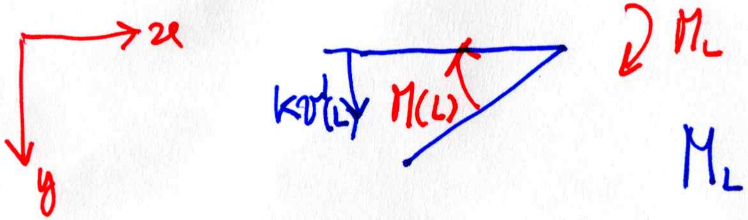


① $v(0) = 1$ ② $v'(0) = 0$

③ $v(L) = 0$

$T(0) = -V_0$ $T(L) = V_L$

$M(0) = M_0$ $M(L) = -M_L$



$$\left.\begin{aligned} M_L &= -k\,v(L) \\ M(L) &= -EJ\,v''(L) \end{aligned}\right\} \Rightarrow \text{④} \quad M_L = -M(L) = EJ\,v''(L)$$

① $v(0) = 1$

② $v'(0) = 0$

③ $v(L) = 0$

④ $EJ\,v''(L) = -k\,v'(L)$; $EJ\,v''(L) + k\,v'(L) = 0$

Ricavo le costanti di integrazione

$v(x) = Ax^3 + Bx^2 + Cx + D$; $v'(x) = 3Ax^2 + 2Bx + C$; $v''(x) = 6Ax + 2B$

$v'''(x) = 6A$

dalla ① $v(0) = 1 = D \Rightarrow D = 1$

dalla ② $v'(0) = 0 = C \Rightarrow C = 0$

dalla ③ $v(L) = 0 \Rightarrow AL^3 + BL^2 + 1 = 0 \Rightarrow A = -\dfrac{(BL^2+1)}{L^3}$

dalla ④ $EJ\,v''(L) + k\,v'(L) = 0 \Rightarrow EJ(6AL + 2B) + k(3AL^2 + 2BL) = 0$

$$A(6EJL + 3kL^2) + B(2EJ + 2kL) = 0$$

$$-\frac{(BL^2+1)}{L^3}(6EJL + 3kL^2) + B(2EJ + 2kL) = 0$$

$$-\underline{6EJB} - \frac{3k}{L} - \underline{\underline{3kLB}} - \frac{6EJ}{L^2} + \underline{2EJB} + \underline{\underline{2kLB}} = 0$$

$$-4EJB - kLB = \frac{6EJ}{L^2} + \frac{3k}{L} \; ; \quad B(4EJ+kL) = -\left(\frac{6EJ}{L^2} + \frac{3k}{L}\right)$$

$$\boxed{B = \frac{-\left(\frac{6EJ}{L^2} + \frac{3k}{L}\right)}{4EJ+kL}} \; ; \quad A = -\frac{(BL^2+1)}{L^3}$$

$$A = -\frac{BL^2}{L^3} - \frac{1}{L^3} = \frac{\left(\frac{6EJ}{L^2} + \frac{3k}{L}\right)L^2}{L^3(4EJ+kL)} - \frac{1}{L^3} = \frac{6EJ+3kL}{L^3(4EJ+kL)} - \frac{1}{L^3}$$

$$= \frac{6EJ+3kL-4EJ-kL}{L^3(4EJ+kL)} = \boxed{\frac{2EJ+2kL}{L^3(4EJ+kL)} = A}$$

$C = 0$

$D = 1$

$$v(x) = \frac{2(EJ+kL)}{(4EJ+kL)L^3}x^3 - \frac{3(2EJ+kL)}{(4EJ+kL)L^2}x^2 + 1$$

$$v''(x) = \frac{12(EJ+kL)}{(4EJ+kL)L^3}x - \frac{6(2EJ+kL)}{(4EJ+kL)L^2} \; ; \quad v'''(x) = \frac{12(EJ+kL)}{(4EJ+kL)L^3}$$

$$V_0 = -T(0) = EJ\,v'''(0) = \frac{12EJ}{L^3}\,\frac{EJ+kL}{4EJ+kL}$$

$$V_L = T(L) = -EJ\,v'''(L) = -\frac{12EJ}{L^3}\,\frac{EJ+kL}{4EJ+kL}$$

$$M_0 = M(0) = -EJ\,v''(0) = +\frac{6EJ}{L^2}\,\frac{2EJ+kL}{4EJ+kL}$$

$$M_L = -M(L) = EJ\,v''(L) = \frac{12EJ}{L^2}\,\frac{EJ+kL}{4EJ+kL} - \frac{6EJ}{L^2}\,\frac{2EJ+kL}{4EJ+kL} =$$

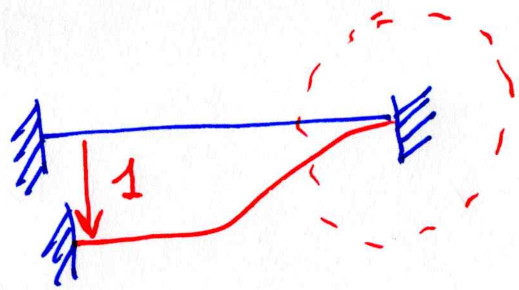
$$= \frac{6EJ}{L^2}\,\frac{\cancel{2EJ}+2kL-\cancel{2EJ}-kL}{4EJ+kL} = \frac{6EJ}{L^2}\,\frac{kL}{4EJ+kL}$$

$$\lim_{k\to\infty}\frac{EJ+kL}{4EJ+kL} = \lim_{k\to\infty}\left(\frac{EJ}{4EJ+kL}\xrightarrow{}0 + \frac{kL}{4EJ+kL}\right) = \lim_{k\to\infty}\frac{kL}{4EJ+kL} = \lim_{k\to\infty}\frac{\cancel{k}L}{\cancel{k}\left(\frac{4EJ}{k}\to 0 + L\right)} = 1$$

$k\to\infty$ (incastro)

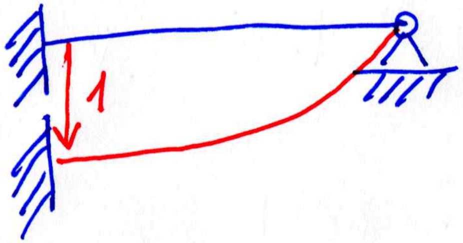
$$\left|\lim_{k\to\infty} V_0\right| = \left|\lim_{k\to\infty} V_L\right| = \frac{12EJ}{L^3}$$

$$\left|\lim_{k\to\infty} M_0\right| = \left|\lim_{k\to\infty} M_L\right| = \frac{6EJ}{L^2}$$



$k\to\infty$

$k \to 0$ (cerniera)

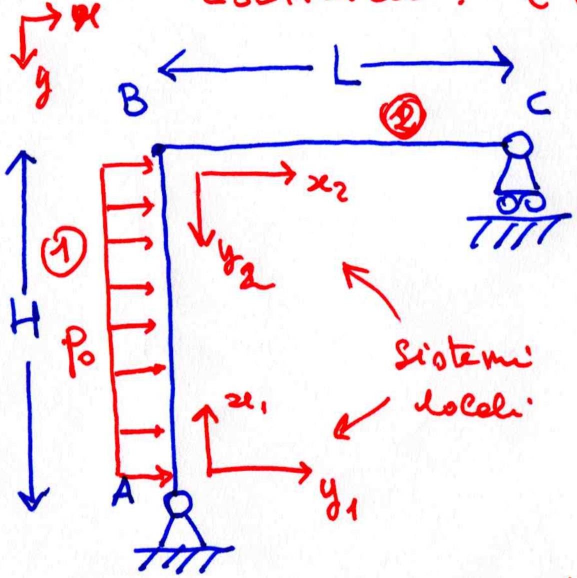


$$\left|\lim_{k\to 0} V_0\right| = \left|\lim_{k\to 0} V_L\right| = \frac{12EJ}{L^3}\cdot\frac{1}{4} = \frac{3EJ}{L^3}$$

$$\lim_{k\to 0} M_0 = \frac{3EJ}{L^2}$$

$$\lim_{k\to 0} M_L = 0$$

ESERCIZIO: TELAIO CON TRAVI DI EULERO-BERNOULLI



$$v_1^{IV}(x_1) = \frac{p_0}{EJ} \qquad v_2^{IV}(x_2) = 0$$

$$v_1(x_1) = \frac{p_0}{24EJ} x_1^4 + A_1 x_1^3 + B_1 x_1^2 + C_1 x_1 + D_1$$

$$v_2(x_2) = A_2 x_2^3 + B_2 x_2^2 + C_2 x_2 + D_2$$

CONDIZIONI AL BORDO

in A

① $v_1(0) = 0$ ② $M_1(0) = 0 \Rightarrow v_1''(0) = 0$

in C

③ $v_2(L) = 0$ ④ $M_2(L) = 0 \Rightarrow v_2''(L) = 0$

CONDIZIONI DI CONTINUITA' IN B

⑤ $v_2(x_2 = 0) = 0$ ⑥ $T_1(x_1 = H) = 0 \Rightarrow v_1'''(H = 0)$

INTEGRO $v_1^{IV}(x_1) = \frac{p_0}{EJ}$

$$v_1'''(x_1) = \frac{p_0}{EJ} x_1 + a_1$$

$$v_1''(x_1) = \frac{p_0}{2EJ} x_1^2 + a_1 x_1 + b_1$$

$$v_1'(x_1) = \frac{p_0}{6EJ} x_1^3 + \frac{a_1}{2} x_1^2 + b_1 x_1 + c_1$$

$$v_1(x_1) = \frac{p_0}{24EJ} x_1^4 + \underbrace{\frac{a_1}{6}}_{A_1} x_1^3 + \underbrace{\frac{b_1}{2}}_{B_1} x_1^2 + \underbrace{c_1}_{C_1} x_1 + \underbrace{d_1}_{D_1}$$

7 $v_1'(x_1 = H) = v_2'(x_2 = 0)$ 8 $M_1(H) = M_2(0) \Rightarrow v_1''(x_1 = H) = v_2''(x_2 = 0)$

DETERMINAZIONE DELLE COSTANTI

$$v_1'(x_1) = \frac{P_0 x_1^3}{6EJ} + 3A_1 x_1^2 + 2B_1 x_1 + C_1 \; ; \; v_1''(x_1) = \frac{P_0 x_1^2}{2EJ} + 6A_1 x_1 + 2B_1$$

$$v_1'''(x_1) = \frac{P_0 x_1}{EJ} + 6A_1$$

$$v_2'(x_2) = 3A_2 x_2^2 + 2B_2 x_2 + C_2 \; ; \; v_2''(x_2) = 6A_2 x_2 + 2B_2 \; ; \; v_2'''(x_2) = 6A_2$$

1 $v_1(0) = 0 \Rightarrow \boxed{D_1 = 0}$ 2 $M_1(0) = 0 \Rightarrow v_1''(0) = 0 \Rightarrow \boxed{B_1 = 0}$

3 $v_2(L) = 0 \Rightarrow \boxed{A_2 L^3 + B_2 L^2 + C_2 L + \cancel{D_2}^{\,0} = 0}$ 4 $M_2(L) = 0 \Rightarrow v_2''(L) = 0 \Rightarrow \boxed{6A_2 L + 2B_2 = 0}$

5 $v_2(0) = 0 \Rightarrow \boxed{D_2 = 0}$

6 $T_1(H) = 0 \Rightarrow v_1'''(H) = 0 \Rightarrow \frac{P_0 H}{EJ} + 6A_1 = 0 \Rightarrow \boxed{A_1 = -\frac{P_0 H}{6EJ}}$

⑦ $v_1'(H) = v_2'(0) \Rightarrow \dfrac{P_0 H^3}{6EJ} + 3\overset{-\frac{P_0 H}{6EJ}}{A_1} H^2 + 2\cancel{B_1}^{\,0} H + C_1 = C_2$

$$\frac{P_0 H^3}{6EJ} - \frac{P_0 H^3}{2EJ} + C_1 = C_2 \; ; \quad \frac{P_0 H^3 - 3P_0 H^3}{6EJ} + C_1 = C_2 \; ;$$

$$\boxed{C_1 = C_2 + \frac{P_0 H^3}{3EJ}}$$

⑧ $M_1(H) = M_2(0) \Rightarrow v_1''(x_1 = H) = v_2''(x_2 = 0) \Rightarrow \dfrac{P_0 H^2}{2EJ} + 6\overset{-\frac{P_0 H}{6EJ}}{A_1} H + 2\cancel{B_1}^{\,0} = 2B_2$

$$\frac{P_0 H^2}{2EJ} - \frac{P_0 H^2}{EJ} = 2B_2 \; ; \quad \boxed{B_2 = -\frac{P_0 H^2}{4EJ}}$$

$$A_1 = -\frac{P_0 H}{6EJ} \; ; \quad B_1 = 0 \; ; \quad C_1 = C_2 + \frac{P_0 H^3}{3EJ} \; ; \quad D_1 = 0$$

$$A_2 L^3 + B_2 L^2 + C_2 L = 0 \; ; \quad 6A_2 L + 2B_2 = 0 \; ; \quad B_2 = -\frac{P_0 H^2}{4EJ} \; ; \quad D_2 = 0$$

$$6A_2L - \cancel{12}\,\frac{p_0H^2}{\cancel{4}_{2}EJ} = 0\ ; \qquad \boxed{A_2 = \frac{p_0H^2}{12EJL}}$$

$$A_2L^3 + B_2L^2 + C_2L = 0\ ; \quad A_2L^2 + B_2L + C_2 = 0\ ; \quad C_2 = -(A_2L^2 + B_2L)$$

$$C_2 = -\left(\frac{p_0H^2L}{12EJ} - \frac{p_0H^2L}{4EJ}\right) = -\frac{p_0H^2L - 3p_0H^2L}{12EJ} = \boxed{\frac{p_0H^2L}{6EJ} = C_2}$$

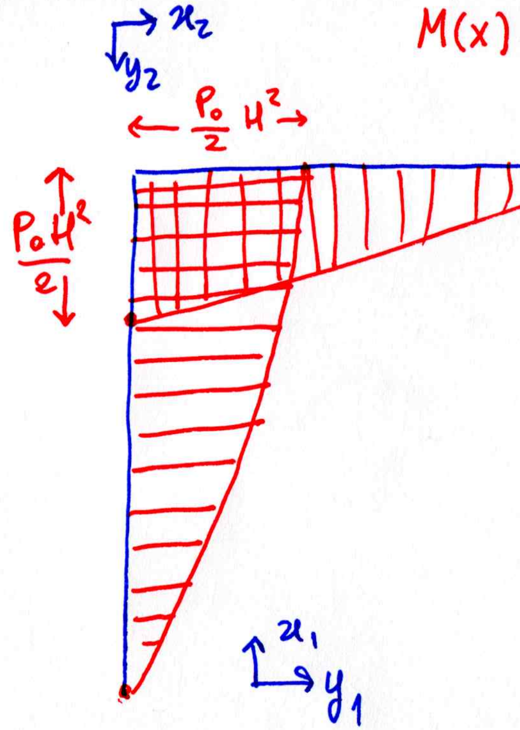
$$C_1 = C_2 + \frac{p_0H^3}{3EJ} = \frac{p_0H^2L}{6EJ} + \frac{p_0H^3}{3EJ} = \boxed{\frac{p_0H^2}{3EJ}\left(\frac{L}{2} + H\right) = C_1}$$

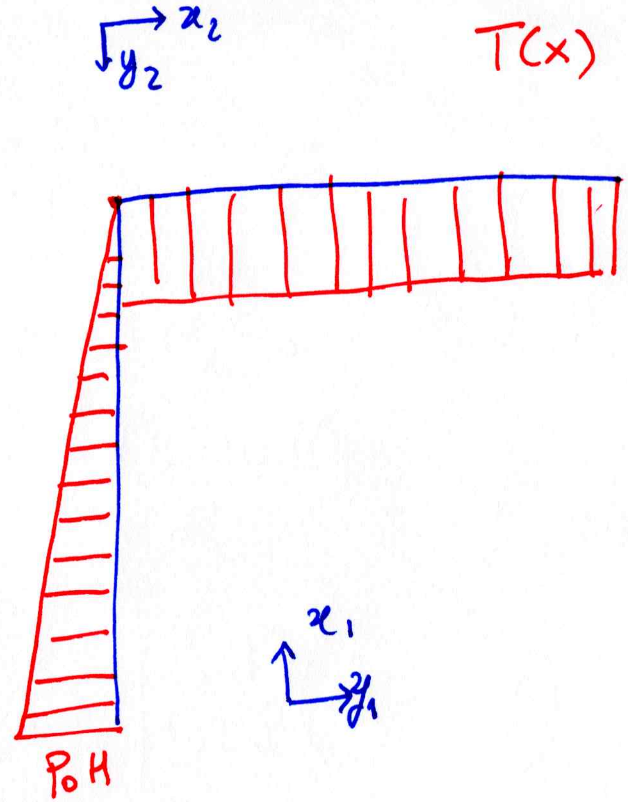
$$M(x) = -EJ\,v''(x) \qquad T(x) = -EJ\,v'''(x)$$

FARE I CONTI COME ESERCIZIO

$$M_1(x_1) = p_0Hx_1 - \frac{p_0}{2}x_1^2 \qquad M_2(x_2) = -\frac{p_0H^2}{2L}x_2 + \frac{p_0H^2}{2}$$

$$T_1(x_1) = p_0H - p_0x_1 \qquad T_2(x_2) = -\frac{p_0H^2}{2L}$$


x_2
y_2
M(x)
$\frac{P_0}{2} H^2$
$\frac{P_0 H^2}{2}$
x_1
y_1


x_2
y_2
T(x)
x_1
y_1
$P_0 H$