Trave su suolo elastico alla Winkler

3 Novembre 2020

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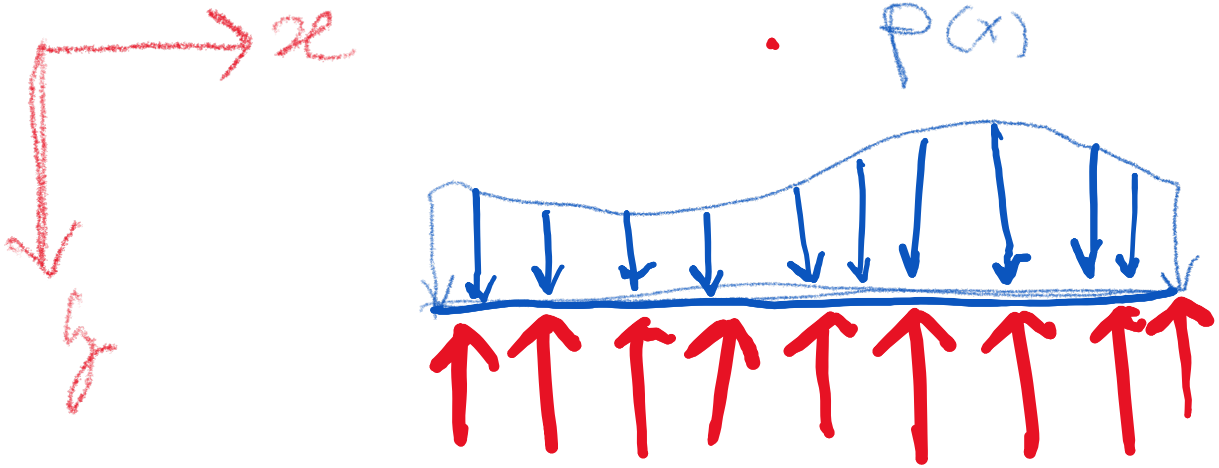
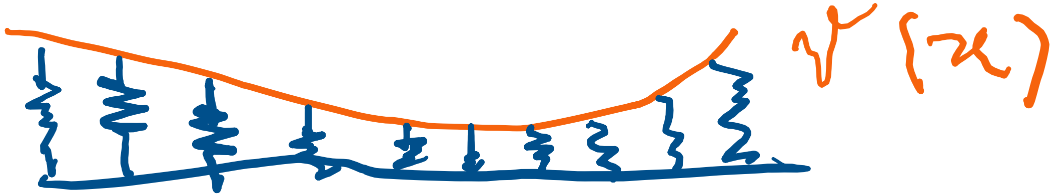


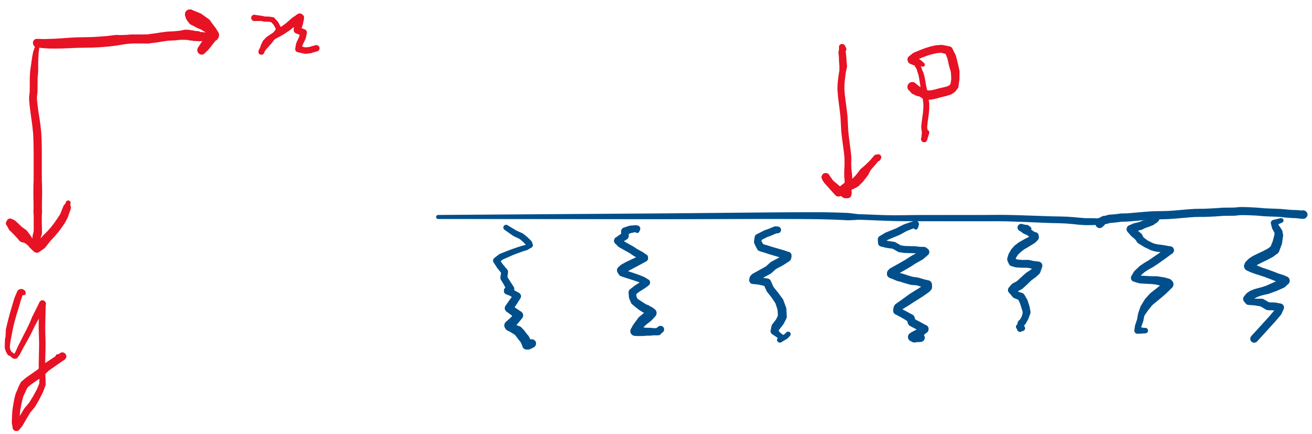
$$\boxed{r(x) = \beta\, v(x)}$$

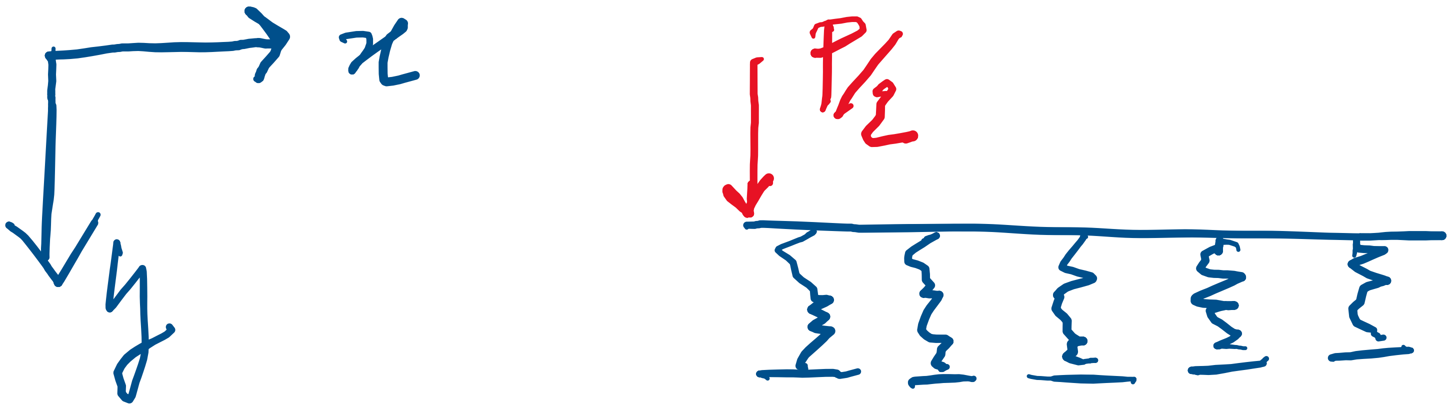


$$EJ\, v^{IV}(x) = p(x) - r(x)$$

$$EJ\, v^{IV}(x) = p(x) - \beta\, v(x)$$

$$EJ\, v^{IV}(x) + \beta\, v(x) = p(x)$$





$$v'(0) = 0$$

$$v^{IV}(x) + \frac{\beta}{EJ} v(x) = 0$$

$$v(x) = c\, e^{\lambda x}$$

$$v(x) = c\, e^{\lambda x}$$

$$v'(x) = \lambda c\, e^{\lambda x}$$

$$v''(x) = \lambda^2 c\, e^{\lambda x}$$

$$v'''(x) = \lambda^3 c\, e^{\lambda x}$$

$$v^{IV}(x) = \lambda^4 c\, e^{\lambda x}$$

$$v^{IV}(x) + \frac{\beta}{EJ} v(x) = 0$$

$$\frac{P}{EJ} = 4\alpha^4 > 0$$

$$v^{IV}(x) + 4\alpha^4 v(x) = 0$$

$$\lambda^4 c\, e^{\lambda x} + 4\alpha^4 c\, e^{\lambda x} = 0$$

$$c\, e^{\lambda x}(\lambda^4 + 4\alpha^4) = 0$$

$$\lambda^4 + 4\alpha^4 = 0$$

$$\lambda^4 = -4\alpha^4$$

$$\lambda^2 = \pm 2\alpha^2 i$$

$$\lambda = \pm \alpha \sqrt{\pm 2i}$$

$$2i = 2i + 1 - 1 = 2i + 1^2 + i^2 = (1+i)^2$$

$$-2i = (1-i)^2$$

$$\lambda = \pm \alpha \sqrt{(1 \pm i)^2}$$

$$= \pm \alpha (1 \pm i)$$

Negli Appunti

$$\lambda_1 = \alpha (1+i) = \lambda_1$$

$$\lambda_2 = \alpha (1-i) = \lambda_3$$

$$\lambda_3 = -\alpha (1+i) = \lambda_2$$

$$\lambda_4 = -\alpha (1-i) = \lambda_4$$

$$v(x) = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x} + C_3 e^{\lambda_3 x} + C_4 e^{\lambda_4 x}$$

$$= C_1 \underline{e}^{\alpha x + i\alpha x} + C_2 \underline{e}^{\alpha x - i\alpha x} + C_3 \underline{\underline{e}}^{-\alpha x - i\alpha x} + C_4 \underline{\underline{e}}^{-\alpha x + i\alpha x}$$

$$v(x) = e^{\alpha x}\left[C_1 e^{i\alpha x} + C_2 e^{-i\alpha x}\right] + e^{-\alpha x}\left[C_3 e^{-i\alpha x} + C_4 e^{i\alpha x}\right]$$

$$e^{\pm i\alpha x} = \cos\alpha x \pm i \sin\alpha x$$

$$v(x) = \left[c_1 (\cos \alpha x + i \sin \alpha x) + c_2 (\cos \alpha x - i \sin \alpha x) \right] e^{\alpha x} + \left[c_3 (\cos \alpha x - i \sin \alpha x) + c_4 (\cos \alpha x + i \sin \alpha x) \right] e^{-\alpha x}$$

$$v(x) = e^{\alpha x}\left(A_1 \sin \alpha x + A_2 \cos \alpha x\right) + e^{-\alpha x}\left(A_3 \sin \alpha x + A_4 \cos \alpha x\right)$$

$x = 0$ ① $v'(0) = 0$

② $T(0) = V_0 = P/2$

$x \to +\infty$ ③ $\lim_{x \to +\infty} v(x) = 0$

③ $\lim_{x \to +\infty} v(x) = 0$

$$\Rightarrow \quad A_1 = A_2 = 0$$

$$v(x) = e^{-\alpha x}\left(A_3 \sin \alpha x + A_4 \cos \alpha x\right)$$

$$v'(x) = -\alpha e^{-\alpha x}\left(A_3 \sin\alpha x + A_4 \cos\alpha x\right) +$$

$$+ e^{-\alpha x}\left(\alpha A_3 \cos\alpha x - \alpha A_4 \sin\alpha x\right)$$

$$= \alpha e^{-\alpha x}\left[-(A_3 + A_4)\sin\alpha x + (A_3 - A_4)\cos\alpha x\right]$$

① $v'(0) = 0$

$$v'(0) = \alpha \overset{1}{e^{0}}\left[-(A_3 + A_4)\overset{0}{\sin 0} + (A_3 - A_4)\overset{1}{\cos 0}\right] = 0$$

$$v'(0)=0 \Rightarrow \alpha(A_3-A_4)=0$$

$$\boxed{A_3=A_4=A}$$

$$v'(x)=-2A\alpha e^{-\alpha x}\sin\alpha x$$

$$v''(x)=2A\alpha^2 e^{-\alpha x}\sin\alpha x-2A\alpha^2 e^{-\alpha x}\cos\alpha x$$

$$v''(x) = 2A\alpha^2 e^{-\alpha x}(\sin\alpha x - \cos\alpha x)$$

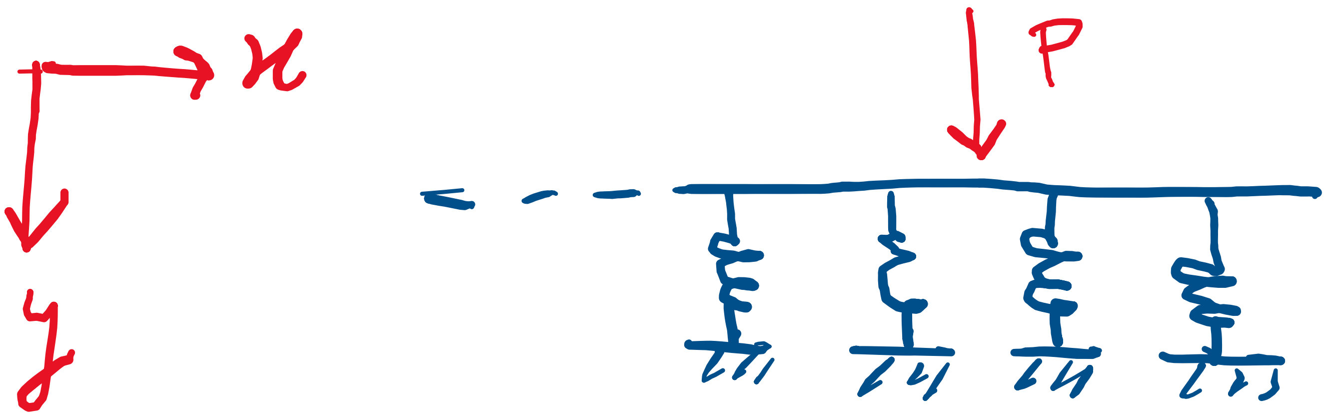
$$v'''(x) = -2A\alpha^3 e^{-\alpha x}(\sin\alpha x - \cos\alpha x)$$
$$+ 2A\alpha^3 e^{-\alpha x}(\cos\alpha x + \sin\alpha x)$$
$$= 2A\alpha^3 e^{-\alpha x}(\cancel{-\sin\alpha x} + \cos\alpha x + \cos\alpha x + \cancel{\sin\alpha x})$$
$$= 4A\alpha^3 e^{-\alpha x}\cos\alpha x$$

2 $\quad T(0) = P/2$

$$-EJ\, v'''(0) = P/2$$

$$-EJ\, 4 A \alpha^3 = P/2 \qquad 4\alpha^4 = \frac{P}{EJ}$$

$$A = -\frac{P}{2} \frac{1}{EJ\, 4\, \alpha^3}$$

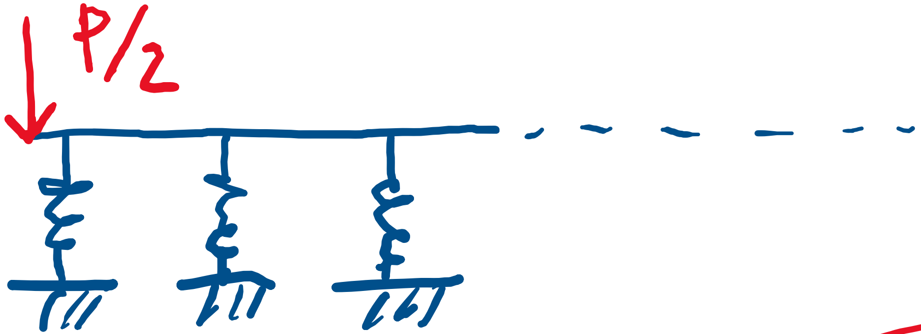


$$r(x) = \beta\, v(x)$$

$$v^{IV}(x) = \frac{p(x)}{EJ} - \frac{r(x)}{EJ}$$

$$\beta = 4\alpha^4$$

$$v^{IV}(x) + \frac{\beta}{EJ}\, v(x) = 0$$



$$v(x) = e^{\alpha x}\left(A_1 \sin \alpha x + A_2 \cos \alpha x\right) + e^{-\alpha x}\left(A_3 \sin \alpha x + A_4 \cos \alpha x\right)$$

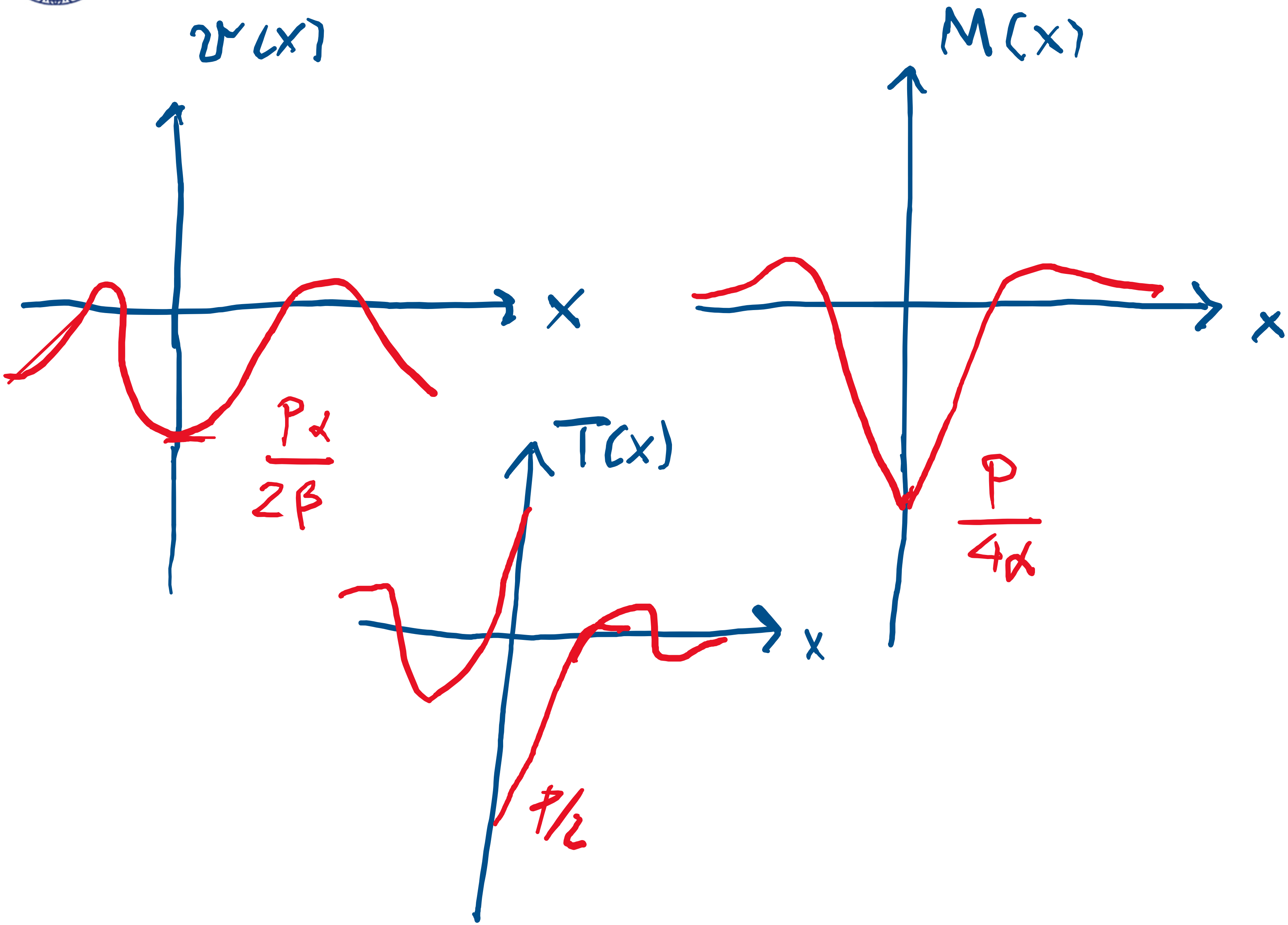
$= 0$ ③ $\lim_{x \to +\infty} v(x) = 0$

① – ② $v'(0) = 0$ $T(0) = P/2$

$$A_3 = A_4 = A = -\frac{P}{2} \frac{1}{EJ4\alpha^3}$$

$$A = \frac{P}{8\alpha^3 EJ} = \frac{P\alpha}{2\cdot 4\alpha^4 EJ} = \frac{P\alpha}{2\beta}$$

$$v(x) = -\frac{P\alpha}{2\beta} e^{-\alpha x} \left(\sin \alpha x + \cos \alpha x \right)$$


v(x)
x
Pα/2β
M(x)
x
P/4α
T(x)
x
P/2

$$\alpha \lambda = 2\pi \qquad \lambda = \frac{2\pi}{\alpha}$$

$$\alpha^4 = \frac{\beta}{4EJ} \Rightarrow \lambda = 2\pi \sqrt[4]{\frac{4EJ}{\beta}}$$

Trave di Timoshenko

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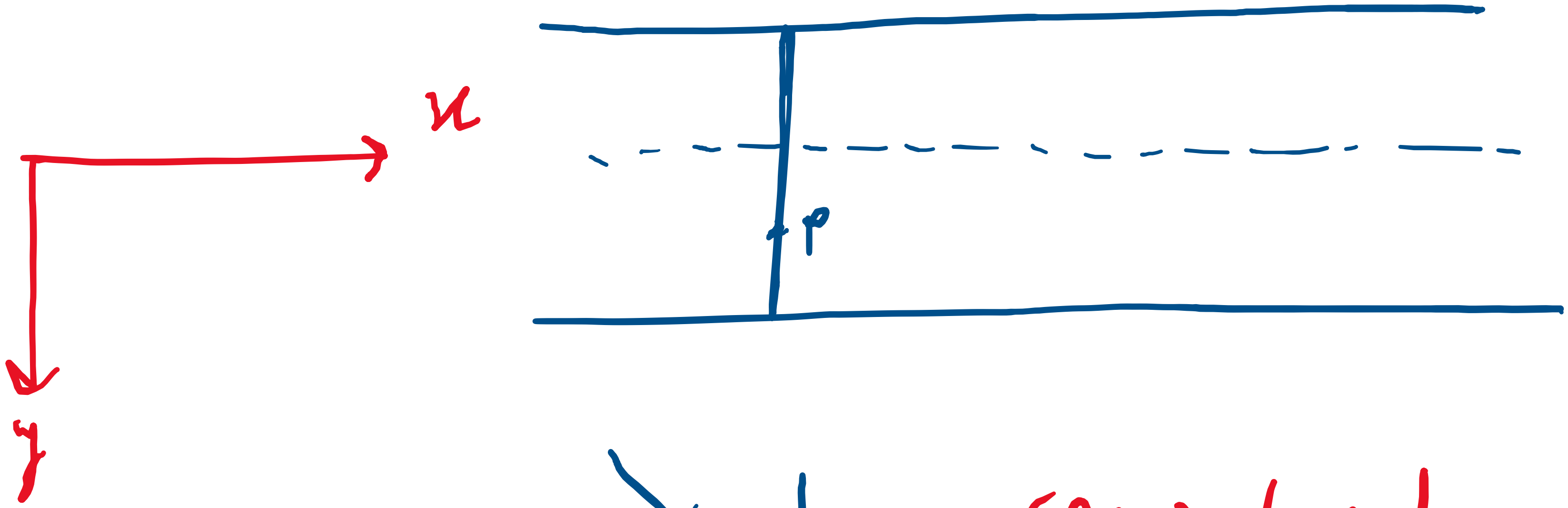
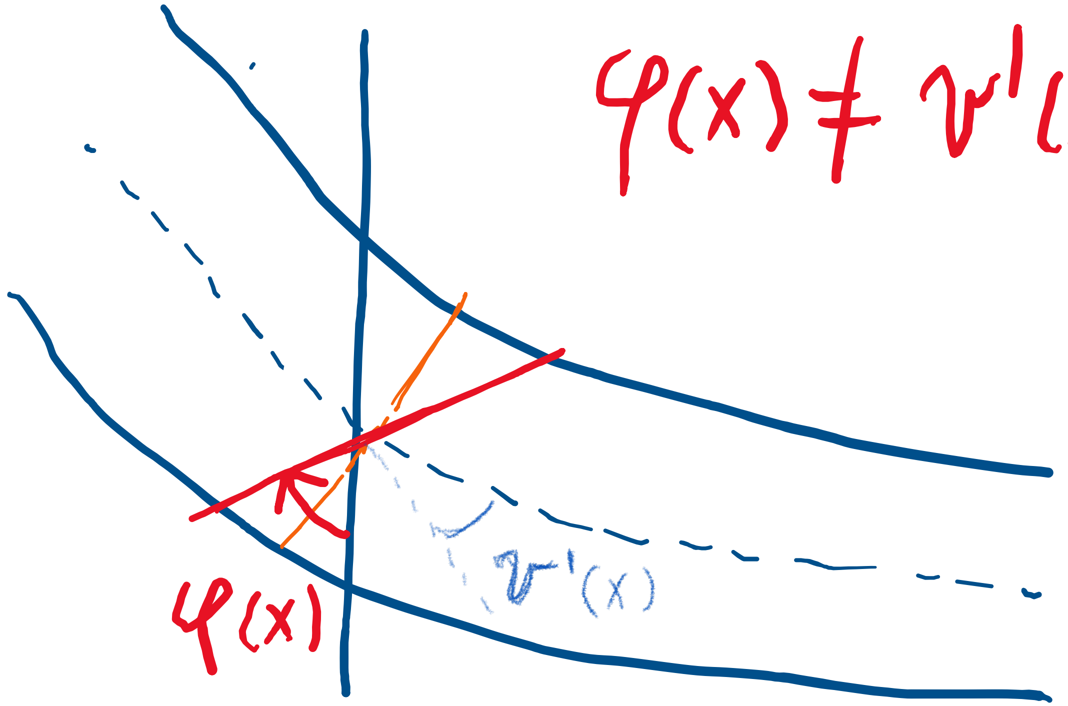


$$\varphi(x) \neq v'(x)$$



$$S_x(x,y) = u(x) - \varphi(x)\,y$$

$$S_y(x,y) = v(x)$$

Deformazioni locali $\varepsilon_x, \varepsilon_y, \gamma_{xy}$

$$\varepsilon_x(x,y) = \frac{\partial S_x}{\partial x} = \frac{\partial u}{\partial x} - y\frac{\partial \varphi}{\partial x}$$

$$\varepsilon_x(x,y) = \frac{\partial S_y}{\partial y} = 0$$

$$\gamma_{xy} = \frac{\partial s_x}{\partial y} + \frac{\partial s_y}{\partial x}$$

$$\boxed{\gamma_{xy} = -\varphi(x) + \frac{dv(x)}{dx} \neq 0}$$

$$\varepsilon_x = \frac{du}{dx} - y\frac{d\varphi}{dx} = \eta(x) + y\chi(x)$$

$$\gamma_{xy} = -\varphi + \frac{dv}{dx} = t(x)$$

deformazione al taglio