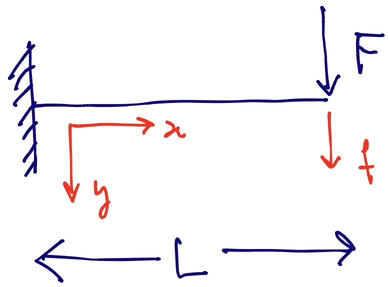
RESUMÉ ESERCIZIO 1 (TIMOŠENKO)



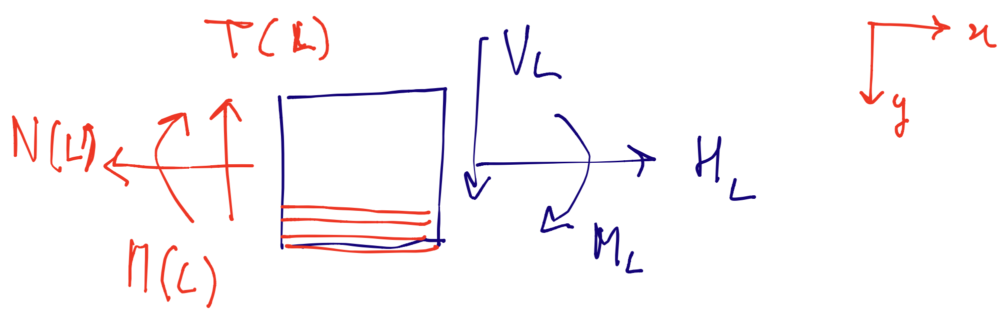
1. $GA_t\left(v''(x)-\varphi'(x)\right)=-p(x)$

2. $-EJ\varphi''(x)-T(x)-m(x)=0$

1. $v(0)=0$ 2. $\varphi(0)=0$

3. $V_L=F=T(L)=GA_t\left(v'(L)-\varphi(L)\right)$

4. $M_L=0=-M(L)=EJ\varphi'(L)=0$



$$v(x)=\frac{Fx}{GA_t}+\frac{Fx^2}{2EJ}\left(L-\frac{x}{3}\right)$$

$$\varphi(x)=\frac{Fx}{EJ}\left(L-\frac{x}{2}\right)$$

$$f = v(x=L) = \frac{FL}{GA_t} + \frac{FL^3}{3EJ}$$

freccia dovuta al taglio

freccia dovuta a flessione

$$f_{flex} = \frac{FL^3}{3EJ}$$

correzione dovuta al taglio

$$f = f_{flex}\left(1 + \frac{FL}{GA_t}\,\frac{3EJ}{FL^3}\right) = f_{flex}\left(1 + \frac{3EJ}{GA_t L^2}\right)$$

IPOTESI SULLE CARATTERISTICHE DELLA TRAVE

$$A = B \times H \; ; \; J = \frac{1}{12} BH^3 \; ; \; A_t = \frac{5}{6} BH$$

$$G = \frac{E}{2(1+\nu)} \; ; \; \nu = \frac{1}{4} \Rightarrow G = \frac{2}{5} E$$

quindi la correzione risulta

$$\frac{3EJ}{GA_t L^2} = \frac{\cancel{3}\,\cancel{E}\,\frac{1}{\cancel{12}_{4}}\,\cancel{B}H^{\cancel{3}\,2}}{\frac{\cancel{2}}{\cancel{5}}\,\cancel{E}\,\frac{\cancel{5}}{\cancel{6}_{3}}\,\cancel{B}\cancel{H}\,L^2} = \frac{3}{4}\,\frac{H^2}{L^2}$$

TIMOSHENKO

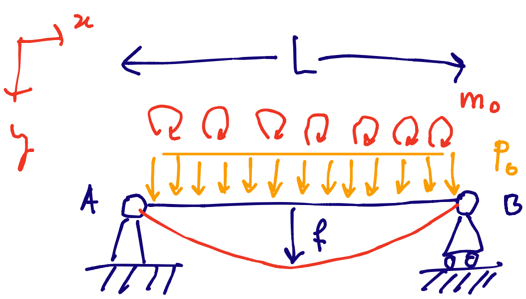
$$\frac{H}{L} = \frac{1}{5} \longrightarrow \frac{3EJ}{GA_t L^2} = \frac{3}{100}$$

$$\frac{H}{L} = \frac{1}{10} \longrightarrow \frac{3EJ}{GA_t L^2} = \frac{3}{400} = 0{,}0075$$

$$\frac{H}{L} = \frac{1}{20} \longrightarrow \frac{3EJ}{GA_t L^2} = 0{,}001875$$

EULERO

ESERCIZIO 2



EQUAZIONI GOVERNANTI

i. $GA_t \left(v''(x) - \varphi'(x) \right) = -p_0$

ii. $EJ\varphi''(x) + GA_t \left(v'(x) - \varphi(x) \right) = -m_0$

CONDIZIONI AL BORDO

1. $v(0) = 0$ 2. $M(0) = 0 \Rightarrow -EJ\varphi'(0) = 0 \Rightarrow \varphi'(0) = 0$

3. $v(L) = 0$ 4. $M(L) = 0 \Rightarrow \varphi'(L) = 0$

Integro la (i)

$$\underbrace{GA_t\left(v'(x)-\varphi(x)\right)}_{\to\ (ii)} = -p_0 x + C_1 \qquad (*)$$

$$EJ\,\varphi''(x) - p_0 x + C_1 = -m_0$$

$$\varphi''(x) = \frac{p_0 x}{EJ} - \frac{m_0}{EJ} - \frac{C_1}{EJ}$$

$$\varphi'(x) = \frac{p_0 x^2}{2EJ} - \frac{m_0 x}{EJ} - \frac{C_1 x}{EJ} + C_2$$

$$\varphi(x) = \frac{p_0 x^3}{6EJ} - \frac{m_0 x^2}{2EJ} - \frac{C_1 x^2}{2EJ} + C_2 x + C_3$$

RICAVO LE COSTANTI

(2) $\varphi'(0) = 0 \Rightarrow \boxed{C_2 = 0}$

(4) $\varphi'(L) = 0 \Rightarrow \dfrac{p_0 L^2}{2EJ} - \dfrac{m_0 L}{EJ} - \dfrac{C_1 L}{EJ} = 0$

$$\frac{L}{EJ}\left(\frac{p_0 L}{2} - m_0 - C_1\right) = 0 \Rightarrow \boxed{C_1 = -m_0 + \frac{p_0 L}{2}}$$

$$\varphi(x) = \frac{p_0 x^3}{6EJ} - \frac{m_0 x^2}{2EJ} - \left(\frac{p_0 L}{2} - m_0\right)\frac{x^2}{2EJ} + C_3$$

(*) $$GA_t\left(v'(x) - \varphi(x)\right) = -p_0 x + \frac{p_0 L}{2} - m_0$$

$$v'(x) - \varphi(x) = -\frac{p_0 x}{GA_t} + \frac{p_0 L}{2GA_t} - \frac{m_0}{GA_t}$$

$$v'(x) = \frac{p_0 x^3}{6EJ} - \cancel{\frac{m_0 x^2}{2EJ}} - \left(\frac{p_0 L}{2} - \cancel{m_0}\right)\frac{x^2}{2EJ} + C_3 - \frac{p_0 x}{GA_t} + \frac{p_0 L}{2GA_t} - \frac{m_0}{GA_t}$$

$$v(x) = \frac{p_0 x^4}{24EJ} - \frac{p_0 L x^3}{12EJ} + C_3 x - \frac{p_0 x^2}{2GA_t} + \frac{p_0 L x}{2GA_t} - \frac{m_0 x}{GA_t} + C_4$$

(1) $v(0) = 0 \Rightarrow C_4 = 0$

(3) $v(L) = 0 \Rightarrow C_3 = \frac{m_0}{GA_t} + \frac{p_0 L^3}{24EJ}$

$$v(x) = \frac{p_0 x^4}{24EJ} - \frac{p_0 x^3 L}{12EJ} + \frac{p_0 x L^3}{24EJ} - \frac{p_0 x^2}{2GA_t} + \frac{p_0 L x}{2GA_t}$$

$$f = v\left(\frac{L}{2}\right) = \underbrace{\frac{5}{384}\frac{p_0 L^4}{EJ}}_{f_{flex}} + \frac{p_0 L^2}{8GA_t}$$

freccia dovute al taglio

$$f = f_{flex}\left(1 + \frac{384}{5}\,\frac{EJ}{\cancel{P_0} \cancel{L^4}\,2}\,\frac{\cancel{P_0}\cancel{L^2}}{8\,G A_t}\right)$$

$$= f_{flex}\left(1 + \frac{48}{5}\,\frac{EJ}{L^2 G A_t}\right)$$

$$A_t = \frac{5}{6} A \quad ; A = BH \; ; \quad \nu = \frac{1}{4} \; ; \quad G = \frac{2}{5} E \; ; J = \frac{BH^3}{12}$$

$$\frac{48}{5}\,\frac{EJ}{L^2 G A_t} = \frac{12}{5}\left(\frac{H}{L}\right)^2$$

$$\frac{H}{L} = \frac{1}{5} \longrightarrow 9{,}6\,\%$$

$$\frac{H}{L} = \frac{1}{10} \longrightarrow 2{,}4\,\%$$

$$\frac{H}{L} = \frac{1}{20} \longrightarrow 0{,}6\,\%$$

$$M(x) = -EJ\varphi'(x)$$

$$T(x) = GA_t\left(v'(x) - \varphi(x)\right)$$

DISEGNARE I DIAGRAMMI DI $M(x)$ e $T(x)$