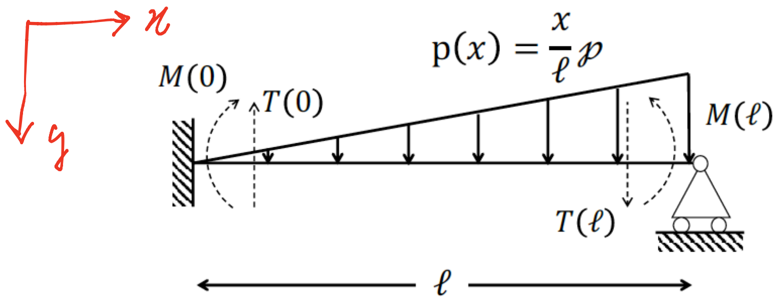
ESERCIZIO SU TRAVE E-B



$$v^{IV}(x) = \frac{p(x)}{EJ} = \frac{p}{EJ\ell}\,x \qquad (*)$$

1 $\quad v(0) = 0 \qquad$ 2 $\quad v'(0) = 0$

3 $\quad v(\ell) = 0 \qquad$ 4 $\quad M(\ell) = 0 \Rightarrow v''(\ell) = 0$

(*)

$$v'''(x) = \frac{p}{2EJ\ell}\,x^2 + C_1$$

$$v''(x) = \frac{p}{6EJ\ell}\,x^3 + C_1 x + C_2$$

$$v'(x) = \frac{p}{24EJ\ell}\,x^4 + \frac{C_1}{2}x^2 + C_2 x + C_3$$

$$v(x) = \frac{p}{120EJ\ell}\,x^5 + \frac{C_1}{6}x^3 + \frac{C_2}{2}x^2 + \cancelto{0}{C_3}\,x + \cancelto{0}{C_4}$$

① $v(0)=0 \Rightarrow C_4=0$

② $v'(0)=0 \Rightarrow C_3=0$

③ $v(l)=0 \Rightarrow \dfrac{P\,l^{\cancel{5}\,4}}{120EJ\cancel{l}}+\dfrac{C_1}{6}l^3+\dfrac{C_2}{2}l^2=0$

$$\boxed{\frac{P\,l^2}{60EJ}+\frac{C_1 l}{3}+C_2=0}$$

④ $v''(l)=0 \Rightarrow v''(x)=\dfrac{P}{6EJl}x^3+C_1x+C_2$

$$\boxed{\frac{P\,l^{\cancel{3}\,2}}{6EJ\cancel{l}}+C_1 l+C_2=0}$$

$$C_2=-C_1 l-\frac{Pl^2}{6EJ}$$

$$\frac{Pl^2}{60EJ}+\frac{C_1 l}{3}-C_1 l-\frac{Pl^2}{6EJ}=0$$

$$-\frac{2}{3} C_1 l + \frac{Pl^2 - 10Pl^2}{60EJ} = 0$$

$$\frac{2}{3} C_1 l + \frac{3pl^2}{20EJ} = 0$$

$$\boxed{C_1 = -\frac{3}{2\cancel{l}} \cdot \frac{3pl^{\cancel{2}}}{20EJ} = -\frac{9}{40}\,\frac{pl}{EJ}}$$

$$C_2 = -C_1 l - \frac{Pl^2}{6EJ} = \frac{9}{40}\,\frac{Pl^2}{EJ} - \frac{Pl^2}{6EJ}$$

$$= \frac{27Pl^2 - 20Pl^2}{120EJ} = \boxed{\frac{7}{120}\,\frac{Pl^2}{EJ} = C_2}$$

$$M(x) = -EJ\,v''(x)$$

$$T(x) = -EJ\,v'''(x)$$

$$v''(x) = \frac{P}{6EJl} x^3 + C_1 x + C_2$$

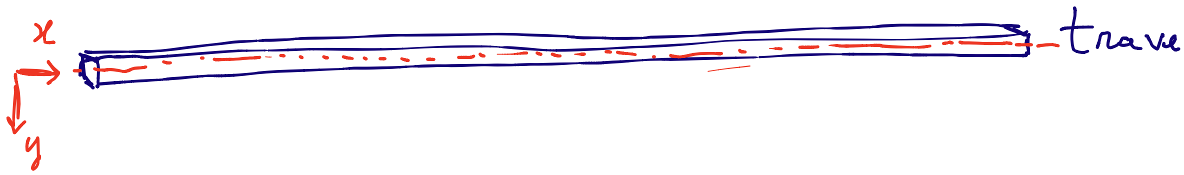
$$= \frac{P}{6EJl} x^3 - \frac{9}{40} \frac{Pl}{EJ} x + \frac{7}{120} \frac{Pl^2}{EJ}$$

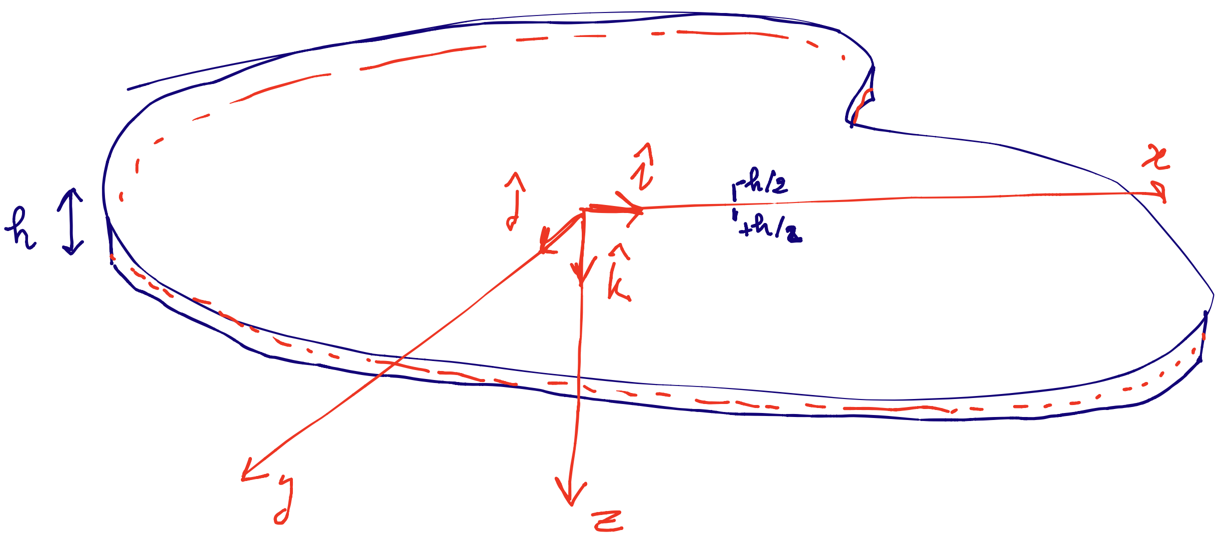
$$M(L) = -EJ\, v''(L)$$

$$= -\frac{Pl^2}{6} + \frac{9}{40} Pl^2 - \frac{7}{120} Pl^2$$

$$= \frac{-20Pl^2 + 27Pl^2 - 7Pl^2}{120} = 0$$

PIASTRE





MODELLO CINEMATICO PER LA PIASTRA DI REISSNER-MINDLIN (DEFORMABILE AL TAGLIO)

NOTA SU TRAVE

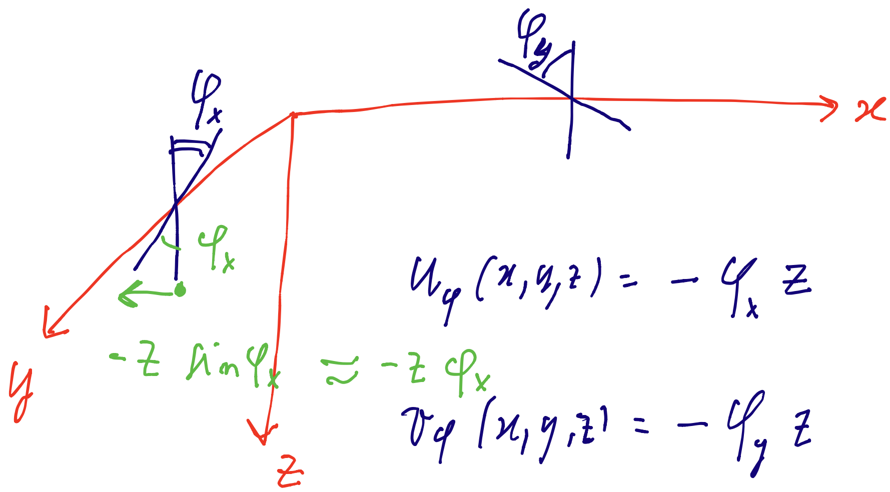
$$s_x = u(x) - y\,\varphi(x)$$

$$s_y = v(x)$$

$$s_x(x,y,z) = u(x,y) \overbrace{- z\,\varphi_x(x,y)}^{u_\varphi}$$

$$s_y(x,y,z) = v(x,y) \underbrace{- z\,\varphi_y(x,y)}_{v_\varphi}$$

$$s_z(x,y,z) = w(x,y)$$



$$u_\varphi(x,y,z) = -\varphi_x\, z$$

$$v_\varphi(x,y,z) = -\varphi_y\, z$$

DEFORMAZIONI $\qquad \varepsilon_z = 0$

$$S_x(x,y,z) = u(x,y) \overbrace{- z\,\varphi_x(x,y)}^{u_\varphi}$$

$$S_y(x,y,z) = v(x,y) \underbrace{- z\,\varphi_y(x,y)}_{v_\varphi}$$

$$S_z(x,y,z) = w(x,y)$$

$$\varepsilon_i = \frac{\partial s_i}{\partial x_i} \qquad \gamma_{ij} = \frac{\partial s_i}{\partial x_j} + \frac{\partial s_j}{\partial x_i}$$

$$\varepsilon_x = \frac{\partial S_x}{\partial x} = \frac{\partial u}{\partial x} - z\frac{\partial \varphi_x}{\partial x}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} - z\frac{\partial \varphi_y}{\partial y}$$

$$\gamma_{xy} = \frac{\partial S_x}{\partial y} + \frac{\partial S_y}{\partial x} = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) - z\left(\frac{\partial \varphi_x}{\partial y} + \frac{\partial \varphi_y}{\partial x}\right)$$

$$\gamma_{xz} = \frac{\partial S_x}{\partial z} + \frac{\partial S_z}{\partial x} = \frac{\partial W}{\partial x} - \varphi_x$$

$$\gamma_{yz} = \frac{\partial W}{\partial y} - \varphi_y$$

$$\begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} 0 & -z & 0 \\ 0 & 0 & -z \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} w \\ \varphi_x \\ \varphi_y \end{pmatrix}$$

$$\vec{S}(x,y,z) = \underset{\sim}{n}_m \vec{u}_m(x,y) + \underset{\sim}{n}_f(z) \vec{u}_f(x,y)$$

membranale flessionale

DEFORMAZIONI GENERALIZZATE

per la trave η, χ, t

$$\eta_x = \frac{\partial u}{\partial x} \; ; \; \eta_y = \frac{\partial v}{\partial y} \; ; \; \eta_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$\chi_x = -\frac{\partial \varphi_x}{\partial x} \; ; \; \chi_y = -\frac{\partial \varphi_y}{\partial y} \; ; \; \chi_{xy} = -\left(\frac{\partial \varphi_x}{\partial y} + \frac{\partial \varphi_y}{\partial x}\right)$$

$$t_x = \frac{\partial w}{\partial x} - \varphi_x \; ; \; t_y = \frac{\partial w}{\partial y} - \varphi_y$$

$$\varepsilon_x = \eta_x(x,y) + z\,\chi_x(x,y) \qquad \gamma_{xz} = t_x(x,y)$$

$$\varepsilon_y = \eta_y(x,y) + z\,\chi_y(x,y) \qquad \gamma_{yz} = t_y(x,y)$$

$$\gamma_{xy} = \eta_{xy}(x,y) + z\,\chi_{xy}(x,y)$$

$$
\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{pmatrix}
=
\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}
\begin{pmatrix} \eta_x \\ \eta_y \\ \eta_{xy} \end{pmatrix}
+
\begin{pmatrix} z & 0 & 0 & 0 & 0 \\ 0 & z & 0 & 0 & 0 \\ 0 & 0 & z & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}
\begin{pmatrix} \chi_x \\ \chi_y \\ \chi_{xy} \\ t_x \\ t_y \end{pmatrix}
$$

$$
\vec{\varepsilon}(x,y,z) = \underbrace{\underline{b}_m \, \vec{q}_m(x,y)}_{\text{membranale}} + \underbrace{\underline{b}_f(z) \, \vec{q}_f(x,y)}_{\text{flessionale}}
$$

$$
\begin{pmatrix} \eta_x \\ \eta_y \\ \eta_{xy} \end{pmatrix}
=
\underbrace{\begin{pmatrix} \partial/\partial x & 0 \\ 0 & \partial/\partial y \\ \partial/\partial y & \partial/\partial x \end{pmatrix}}_{\partial_m(\cdot)}
\begin{pmatrix} u \\ v \end{pmatrix}
\Rightarrow \vec{q}_m \; \partial_m(\vec{u}_m)
$$

$$
\begin{pmatrix} \chi_x \\ \chi_y \\ \chi_{xy} \\ t_x \\ t_y \end{pmatrix}
=
\overbrace{\begin{pmatrix} 0 & -\partial/\partial x & 0 \\ 0 & 0 & -\partial/\partial y \\ 0 & -\partial/\partial y & -\partial/\partial x \\ \partial/\partial x & -1 & 0 \\ \partial/\partial y & 0 & -1 \end{pmatrix}}^{\partial_f(\cdot)}
\begin{pmatrix} w \\ \varphi_x \\ \varphi_y \end{pmatrix}
\Rightarrow \vec{q}_f = \partial_f(\vec{u}_f)
$$

PRINCIPIO DEI LAVORI VIRTUALI (1)

↳ Introduzione delle forze generalizzate

$$\vec{F}\cdot\vec{S}\ , \quad \vec{F}^t\vec{S}$$

LAVORO ESTERNO

$$\vec{F}(x,y,z) = \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} \qquad \vec{F}^t = (F_x, F_y, F_z)$$

$$\frac{d\mathcal{L}_e}{dA} = \int_{-h/2}^{h/2} \vec{F}^t \delta\vec{S}\, dz = \left[\int_{-h/2}^{h/2} \vec{F}^t \underline{n}_m\, dz\right]\cdot \delta\hat{\vec{U}}_m + \left[\int_{-h/2}^{h/2} \vec{F}^t \underline{n}_f(z)\, dz\right]\cdot\delta\hat{\vec{U}}_f$$

$$\boxed{\vec{S}(x,y,z) = \underline{n}_m \vec{U}_m(x,y) + \underline{n}_f(z)\vec{U}_f(x,y)}$$

$$= \vec{P}_m^t\, \delta\hat{\vec{U}}_m + \vec{P}_f^t\, \delta\hat{\vec{U}}_f$$

RICORDIAMO CHE:

$$\underline{n}_m = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad \underline{n}_f(z) = \begin{pmatrix} 0 & -z & 0 \\ 0 & 0 & -z \\ 1 & 0 & 0 \end{pmatrix}$$

$$\vec{F}^t \underline{n}_m = (F_x, F_y, F_z)\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} = (F_x, F_y)$$

$$\vec{F}^t \underline{n}_f(z) = (F_x, F_y, F_z)\begin{pmatrix} 0 & -z & 0 \\ 0 & 0 & -z \\ 1 & 0 & 0 \end{pmatrix} = (F_z, -zF_x, -zF_y)$$

$$\vec{P}_m^t = \int_{-h/2}^{h/2} (F_x, F_y)\, dz \Rightarrow \vec{P}_m = \int_{-h/2}^{h/2} \begin{pmatrix} F_x \\ F_y \end{pmatrix} dz = \begin{pmatrix} n_x \\ n_y \end{pmatrix}$$

$$\vec{P}_f = \int_{-h/2}^{h/2} \begin{pmatrix} F_z \\ -z F_x \\ -z F_y \end{pmatrix} dz = \begin{pmatrix} p \\ m_x \\ m_y \end{pmatrix}$$

$$n_x = \int_{-h/2}^{h/2} F_x \, dz \quad ; \quad n_y = \int_{-h/2}^{h/2} F_y \, dz \quad ; \quad p = \int_{-h/2}^{h/2} F_z \, dz$$

$$m_x = -\int_{-h/2}^{h/2} z F_x \, dz \quad ; \quad m_y = -\int_{-h/2}^{h/2} z F_y \, dz$$

LAVORO INTERNO (SFORZI GENERALIZZATI)

$$\vec{\sigma} = \begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{pmatrix} \qquad \frac{d\mathcal{L}_i}{dA} = \int_{-h/2}^{h/2} \vec{\sigma}^t \, \delta \hat{\vec{\varepsilon}} \, dz$$

$$\vec{\varepsilon}(x,y,z) = \underbrace{\underline{b}_m \, \vec{q}_m(x,y)}_{\text{membranale}} + \underbrace{\underline{b}_f(z) \, \vec{q}_f(x,y)}_{\text{flessionale}}$$

$$= \left[\int_{-h/2}^{h/2} \vec{\sigma}^t \, \underline{b}_m \, dz \right] \cdot \delta \hat{\vec{q}}_m(x,y) + \left[\int_{-h/2}^{h/2} \vec{\sigma}^t \, \underline{b}_f(z) \, dz \right] \cdot \delta \hat{\vec{q}}_f(x,y)$$

$$= \vec{Q}_m^t \, \delta \hat{\vec{q}}_m + \vec{Q}_f^t \, \delta \hat{\vec{q}}_f$$

$$\vec{Q}_m^t = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}, \tau_{xz}, \tau_{yz}) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} dz = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}) \, dz$$

$$\vec{Q}_m = \begin{pmatrix} \int_{-h/2}^{h/2} \sigma_x \, dz \\ \int_{-h/2}^{h/2} \sigma_y \, dz \\ \int_{-h/2}^{h/2} \tau_{xy} \, dz \end{pmatrix} = \begin{pmatrix} N_x \\ N_y \\ N_{xy} \end{pmatrix}$$

$$\vec{Q}_f^{\,t} = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}, \tau_{xz}, \tau_{yz}) \begin{pmatrix} z & 0 & 0 & 0 & 0 \\ 0 & z & 0 & 0 & 0 \\ 0 & 0 & z & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} dz$$

$$= \int_{-h/2}^{h/2} (z\sigma_x, z\sigma_y, z\tau_{xy}, \tau_{xz}, \tau_{yz}) \, dz = (\underbrace{M_x, M_y, M_{xy}}_{\text{momento}}, \underbrace{T_{xz}, T_{yz}}_{\text{taglio}})$$