PIASTRA DI REISSNER-MINDLIN: IMPOSTIAMO IL PLV

$$\vec{F} = \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} \quad \text{forze esterne} \quad \Rightarrow \quad \text{lavoro virtuale esterno}$$

LAVORO ESTERNO

$$\frac{d\mathcal{L}_e}{dA} = \int_{-h/2}^{h/2} \vec{F}^t \delta \hat{\vec{S}}\, dz = \int_{-h/2}^{h/2} \vec{F} \cdot \delta \hat{\vec{S}}\, dz = \vec{P}_m^t\, \delta \vec{U}_m + \vec{P}_f\, \delta \vec{U}_f$$

$$\vec{P}_m = \begin{pmatrix} n_x \\ n_y \end{pmatrix} \qquad n_x = \int_{-h/2}^{h/2} F_x\, dz \quad ; \quad n_y = \int_{-h/2}^{h/2} F_y\, dz$$

$$\vec{P}_f = \begin{pmatrix} p \\ m_x \\ m_y \end{pmatrix} \qquad p = \int_{-h/2}^{h/2} F_z\, dz \; ; \qquad m_x = \int_{-h/2}^{h/2} -z\, F_x\, dz \; ; \quad m_y = \int_{-h/2}^{h/2} -z F_y\, dz$$

LAVORO INTERNO

$$\vec{\sigma} = \begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{pmatrix} \qquad \frac{d\mathcal{L}_i}{dA} = \int_{-h/2}^{h/2} \vec{\sigma}^t\, \delta \hat{\vec{\varepsilon}}\, dz$$

$$\hookrightarrow \vec{\varepsilon} = \underline{b}_m\, \vec{q}_m + \underline{b}_f\, \vec{q}_f$$

$$= \left[\int_{-h/2}^{h/2} \vec{\sigma}^t\, \underline{\underline{b}}_m\, dz \right] \cdot \delta \vec{q}_m + \left[\int_{-h/2}^{h/2} \vec{\sigma}^t\, \underline{b}_f\, dz \right] \cdot \delta \vec{q}_f$$

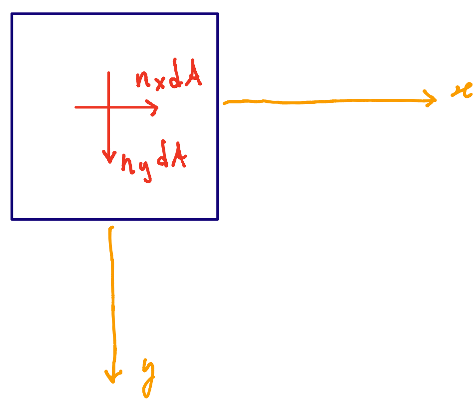
$$= \vec{Q}_m^t\, \delta \vec{q}_m + \vec{Q}_f^t\, \delta \vec{q}_f$$

$$\vec{Q}_m^t = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy})\, dz = (N_x, N_y, N_{xy})$$

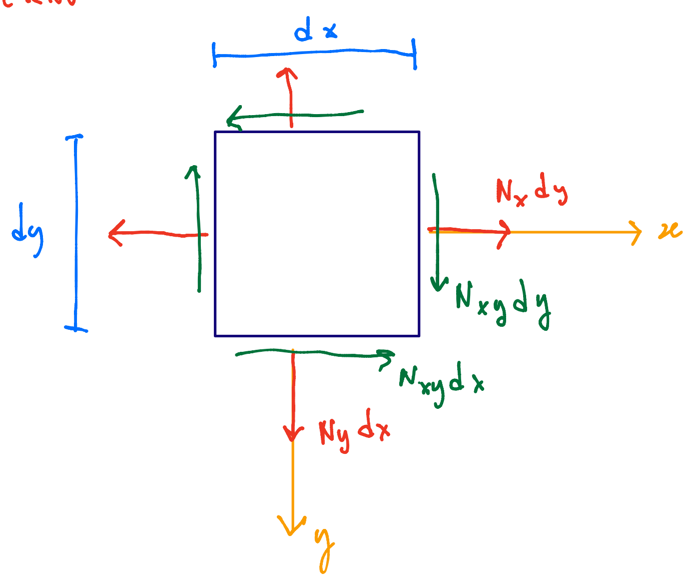
$$\vec{Q}_f^t = \int_{-h/2}^{h/2} (\sigma_x z, \sigma_y z, \tau_{xy} z, \tau_{xz}, \tau_{yz})\, dz = (M_x, M_y, M_{xy}, T_{xz}, T_{yz})$$

AZIONI MEMBRANALI

- AZIONI ESTERNE

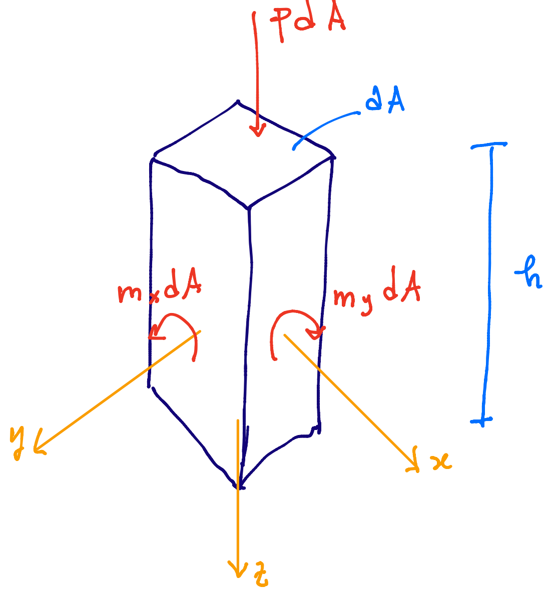


AZIONI INTERNE

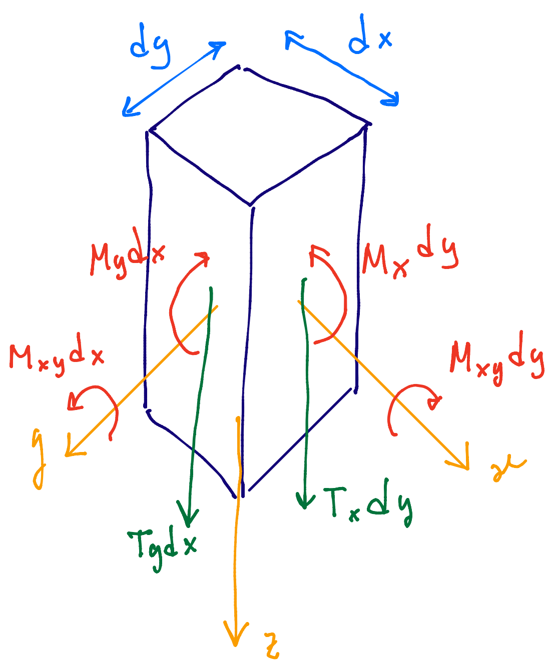


AZIONI FLESSIONALI

• AZIONI ESTERNE



• AZIONI INTERNE



INTEGRAZIONE PER PARTI Leibnitz rule: $D[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$

$$\int_{x_0}^{x_1} \frac{d}{dx}[f(x)\cdot g(x)]\,dx = \int_{x_0}^{x_1} f'(x)\,g(x)\,dx + \int_{x_0}^{x_1} f(x)\,g'(x)\,dx$$

$$\hookrightarrow \int_{x_0}^{x_1} f'(x)\,g(x)\,dx = f(x)\,g(x)\Big|_{x_0}^{x_1} - \int_{x_0}^{x_1} f(x)\,g'(x)\,dx$$

LA SUA GENERALIZZAZIONE A n DIMENSIONI E' IL TEOREMA DELLA DIVERGENZA

$\Omega \subseteq \mathbb{R}^n$ con bordo $\partial\Omega$, Ω aperto e limitato

u, v differenziabili con continuità sulla chiusura di Ω

$\vec{n}$ normale alla superficie $\partial\Omega$

$$\int_{\Omega} \frac{\partial u}{\partial x_i}\, v\, d\Omega = \int_{\partial\Omega} u\, v\, n_i\, d\sigma - \int_{\Omega} u\, \frac{\partial v}{\partial x_i}\, d\Omega$$

in forma vettoriale corrisponde a

$$\int_{\Omega} \nabla u \cdot \vec{v}\, d\Omega = \int_{\partial\Omega} u\,\vec{v}\cdot\vec{n}\, d\sigma - \int_{\Omega} u\, \nabla\cdot\vec{v}\, d\Omega$$

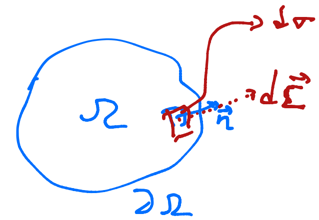
se prendiamo $u = 1 \Rightarrow \nabla 1 = 0$ allora

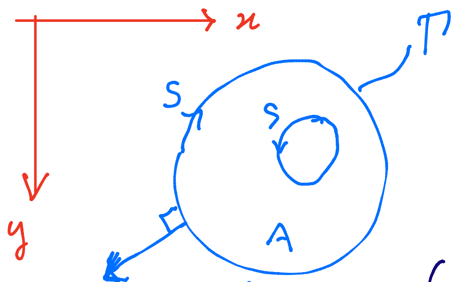
$$0 = \int_{\partial\Omega} \vec{v}\cdot\vec{n}\, d\sigma - \int_{\Omega} \nabla\cdot\vec{v}\, d\Omega$$

$$\boxed{\int_{\Omega} \nabla\cdot\vec{v}\, d\Omega = \int_{\partial\Omega} \vec{v}\cdot\underbrace{\vec{n}\, d\sigma}}$$

TEOREMA DELLA DIVERGENZA

$$\vec{n}\, d\sigma = d\vec{\Sigma}$$





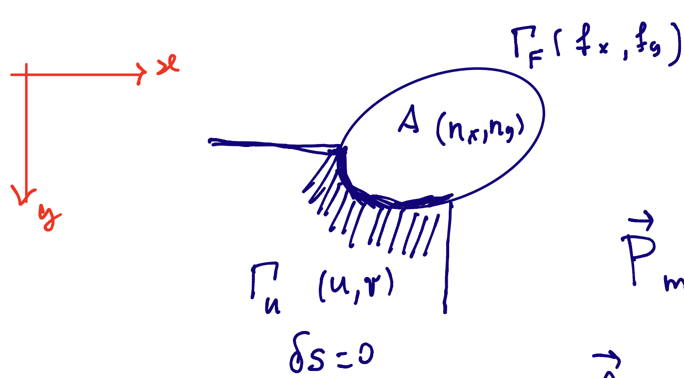
$$\vec{n} = \begin{pmatrix} \alpha_x \\ \alpha_y \end{pmatrix}$$

$$\vec{g} = \begin{pmatrix} g_x \\ g_y \end{pmatrix}$$

$$\int_A \left(\frac{\partial g_x}{\partial x} + \frac{\partial g_y}{\partial y} \right) dA = \int_{\Gamma} \left(g_x \alpha_x + g_y \alpha_y \right) dS$$

$$\int_A \partial_i g_i \, dA = \int_{\Gamma} g_i \, dS_i$$

PROBLEMA MEMBRANALE



$$\delta s \neq 0$$

$$\vec{P}_m = \begin{pmatrix} n_x \\ n_y \end{pmatrix}$$ forze generalizzate membranali

$$\vec{f}_m = \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

$$\mathcal{L}_e = \int_A (n_x \delta u + n_y \delta v)\, dA + \int_{\Gamma_F} (t_x \delta u + t_y \delta v)\, ds$$

$$\delta\eta_x = \delta \frac{\partial u}{\partial x}$$

$$\mathcal{L}_i = \int_A (N_x \delta\eta_x + N_y \delta\eta_y + N_{xy}\delta\eta_{xy})\, dA \qquad \delta\eta_y = \delta \frac{\partial v}{\partial y}$$

$$\delta\eta_{xy} = \delta\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

Sostituendo

$$\mathcal{L}_i = \int_A \left[N_x \frac{\partial \delta u}{\partial x} + N_y \frac{\partial \delta v}{\partial y} + N_{xy}\left(\frac{\partial \delta u}{\partial y} + \frac{\partial \delta v}{\partial x}\right)\right] dA$$

$$= \boxed{\int_A \left[\frac{\partial}{\partial x}\left(N_x \delta u + N_{xy}\delta v\right) + \frac{\partial}{\partial y}\left(N_y \delta v + N_{xy}\delta u\right)\right] dA} +$$

Come si tratta?

$$- \int_A \left(\frac{\partial N_x}{\partial x}\delta u + \frac{\partial N_y}{\partial y}\delta v + \frac{\partial N_{xy}}{\partial x}\delta v + \frac{\partial N_{xy}}{\partial y}\delta u\right) dA$$

$$N_x \frac{\partial \delta u}{\partial x} = \frac{\partial}{\partial x}(N_x \delta u) - \frac{\partial N_x}{\partial x}\delta u \qquad \int_A \left(\frac{\partial g_x}{\partial x} + \frac{\partial g_y}{\partial y}\right) dA = \int_\Gamma (g_x \alpha_x + g_y \alpha_y)\, ds$$

$$\int_A \left[\frac{\partial}{\partial x}\underbrace{\left(N_x \delta u + N_{xy}\delta v\right)}_{g_x} + \frac{\partial}{\partial y}\underbrace{\left(N_y \delta v + N_{xy}\delta u\right)}_{g_y}\right] dA$$

$$= \int_\Gamma \left(N_x \alpha_x \delta u + N_{xy}\alpha_x \delta v + N_y \alpha_y \delta v + N_{xy}\alpha_y \delta u\right) dA$$

$$\mathcal{L}_i = -\int_A \left[\left(\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} \right) \delta u + \left(\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} \right) \delta v \right] dA \; +$$

$$+ \int_\Gamma \left[\left(N_x \alpha_x + N_{xy} \alpha_y \right) \delta u + \left(N_{xy} \alpha_x + N_y \alpha_y \right) \delta v \right] dS$$

$$\mathcal{L}_e = \int_A \left(n_x \delta u + n_y \delta v \right) dA + \int_\Gamma \left(t_x \delta u + t_y \delta v \right) dS$$

$\Gamma \rightarrow$ sarebbe Γ_F, ma su Γ_u
$\delta u = 0$ e $\delta v = 0$

$$\mathcal{L}_i = \mathcal{L}_e$$

$$\int_A \left(n_x \delta u + n_y \delta v \right) dA + \int_A \left[\left(\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} \right) \delta u + \left(\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} \right) \delta v \right] dA \; +$$

$$+ \int_\Gamma \left(t_x \delta u + t_y \delta v \right) dS - \int_\Gamma \left[\left(N_x \alpha_x + N_{xy} \alpha_y \right) \delta u + \left(N_{xy} \alpha_x + N_y \alpha_y \right) \delta v \right] dS$$

$$= 0$$

$$\int_A \left[\left(\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} + n_x \right) \delta u + \left(\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} + n_y \right) \delta v \right] dA \; +$$

$$- \int_\Gamma \left[\left(N_x \alpha_x + N_{xy} \alpha_y - t_x \right) \delta u + \left(N_{xy} \alpha_x + N_y \alpha_y - t_y \right) \delta v \right] dS = 0$$

PER L'ARBITRARIETA' DELLE VARIAZIONI δu e δv

EQUAZIONI D'EQUILIBRIO (COMPORTAMENTO NEL PIANO)

$$\left.\begin{aligned} &\boxed{\frac{\partial N_x}{\partial x}} + \frac{\partial N_{xy}}{\partial y} + n_x = 0 \\ &\frac{\partial N_{xy}}{\partial x} + \boxed{\frac{\partial N_y}{\partial y}} + n_y = 0 \end{aligned}\right\} \text{ in } A$$

NOTA DALLA MECCANICA DEL CONTINUO

$$\boxed{\frac{\partial \sigma_x}{\partial x}} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + F_x = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \boxed{\frac{\partial \sigma_y}{\partial y}} + \frac{\partial \tau_{zy}}{\partial z} + F_y = 0$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \boxed{\frac{\partial \sigma_z}{\partial z}} + F_z = 0$$

$$\left.\begin{aligned} &\delta u = 0 \quad \text{oppure} \quad N_x \alpha_x + N_{xy}\alpha_y = t_x \\ &\delta v = 0 \quad \text{oppure} \quad N_{xy}\alpha_x + N_y \alpha_y = t_y \end{aligned}\right\} \text{ su } \Gamma$$

IN MECCANICA DEL CONTINUO

$$\vec{\sigma}_1 n_{\alpha 1} + \vec{\sigma}_2 n_{\alpha 2} + \vec{\sigma}_3 n_{\alpha 3} = \begin{cases} \vec{f} & \text{su } S_F \\ \vec{r} & \text{su } S_u \end{cases}$$