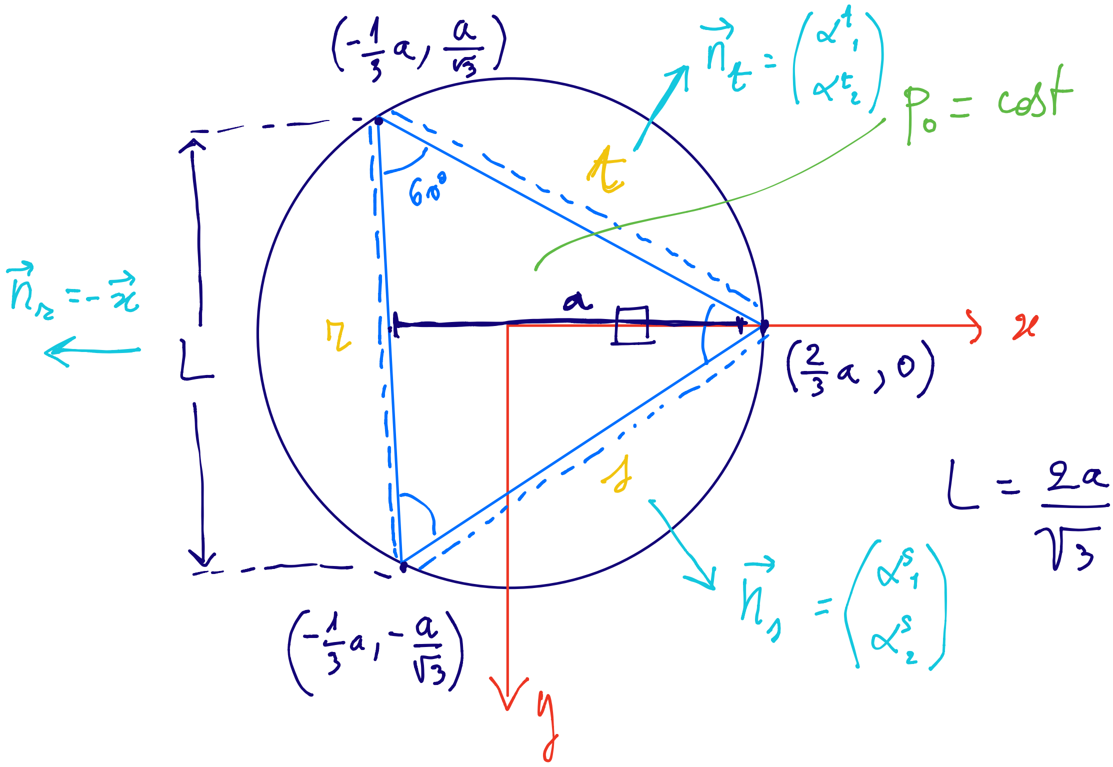


$$W(x,y) = C_0 \left(x + \frac{1}{3}a\right)\left(x - \frac{2}{3}a + y\sqrt{3}\right)\left(x + \frac{2}{3}a - y\sqrt{3}\right)\left(\frac{4}{9}a^2 - x^2 - y^2\right)$$

$$\nabla_4 W = \frac{P}{D}$$

$$\nabla_4 W = \frac{\partial^4 W}{\partial x^4} + 2\frac{\partial^4 W}{\partial x^2 \partial y^2} + \frac{\partial^4 W}{\partial y^4}$$

$$W(x,y) = C_0 \Big(-x^5 + a x^4 + \frac{4}{9}a^2 x^3 - \frac{16}{27}a^3 x^2 + 3y^4 x + 2a x^2 y^2 + 2y^2 x^3 +$$
$$- \frac{4}{3}a^2 y^2 x + a y^4 - \frac{16}{27}a^3 y^2 + \frac{16}{243}a^5\Big)$$

$$\frac{\partial^4 W}{\partial x^4} = C_0(-120x + 24a) \quad ; \quad \frac{\partial^4 W}{\partial y^4} = C_0(72x + 24a)$$

$$\frac{\partial^4 W}{\partial x^2 \partial y^2} = C_0(24x + 8a)$$

$$\nabla_4 W = C_0(24a - \cancel{120x} + 16a + \cancel{48x} + \cancel{72x} + 24a) = \frac{P_0}{D}$$

$$C_0\, 64a = \frac{P_0}{D} \Rightarrow \boxed{C_0 = \frac{P_0}{64aD}}$$

$$W(x,y) = \frac{P_0}{64aD}\left(x + \frac{1}{3}a\right)\left(x - \frac{2}{3}a + y\sqrt{3}\right)\left(x + \frac{2}{3}a - y\sqrt{3}\right)\left(\frac{4}{9}a^2 - x^2 - y^2\right)$$

$$W_{n_1,n_1,t} = 0 \qquad M_{n_{n_1,n_1,t}} = 0$$

su 1) $x = -\frac{1}{3}a \Rightarrow W\left(x = -\frac{1}{3}a\right) = 0$

$$M_x\left(x = -\frac{1}{3}a\right) = -D\left(\frac{\partial^2 W}{\partial x^2} + \nu \frac{\partial^2 W}{\partial y^2}\right)\Bigg|_{x = -\frac{1}{3}a}$$

Come esercizio calcolare $M_x\left(x = -\frac{1}{3}a\right)$

$$W_{max} = W(x=0, y=0) = -\frac{P_0}{64aD}\,\frac{1}{3}a \cdot \frac{4}{9}a^2\,\frac{4}{9}a^2$$

$$= -\frac{P_0 a^4}{972D}$$

$$[P_0] = \frac{[F]}{[L^2]} \qquad [a^4] = [L^4] \qquad [D] = [E h^3]$$

$$= \frac{[F]}{[L^2]}[L^3]$$

$$= [F][L]$$

$$\frac{[F]}{[L^2]} \frac{[L^4]}{[F][L]} = [L]$$

MOMENTI FLETTENTI E TORCENTI

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right) \qquad M_{xy} = -D(1-\nu)\frac{\partial^2 w}{\partial x \partial y}$$

$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right)$$

Calcolo M_x, M_y e M_{xy} lungo una bisettrice $\boxed{y=0}$

$$M_x = \frac{P_0}{16a}\Big[(5-\nu)x^3 - (3+\nu)ax^2 - \frac{2}{3}a^2(1-\nu)x - 3(1+3\nu)xy^2 + \\ - (1+3\nu)ay^2 + \frac{8}{27}(1+\nu)a^3\Big]$$

$$M_y = \frac{P_0}{16a}\Big[(5\nu-1)x^3 - (1+3\nu)ax^2 + \frac{2}{3}a^2(1-\nu)x - 3(3+\nu)xy^2 + \\ - (3+\nu)ay^2 + \frac{8}{27}(1+\nu)a^3\Big]$$

$$M_{xy} = \frac{P_0}{32a}(1-\nu)\Big[6y^3 + 8axy + 6yx^2 - \frac{4}{3}a^2y\Big]$$

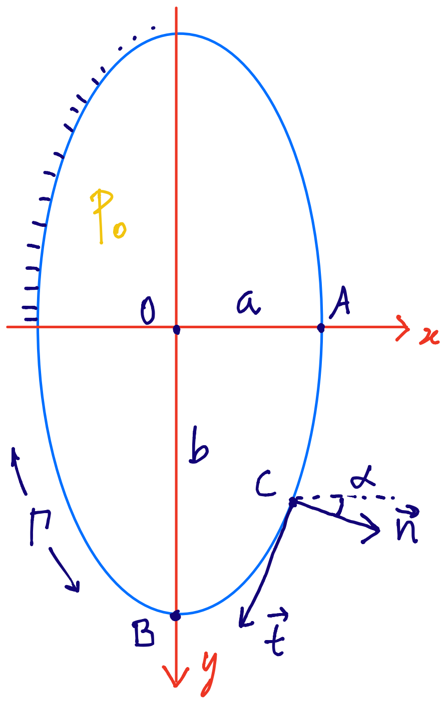
$$M_{xy}(y=0) = 0$$

sulla bisettrice $y=0$ il momento torcente è nullo

$$M_x^{(y=0)} = \frac{p_0}{16a}\Big[(5-\nu)x^3-(3+\nu)ax^2-\frac{2}{3}a^2(1-\nu)x-\cancel{3(1+3\nu)xy^2}+$$
$$-\cancel{(1+3\nu)ay^2}+\frac{8}{27}(1+\nu)a^3\Big]$$

$$M_y^{(y=0)} = \frac{p_0}{16a}\Big[(5\nu-1)x^3-(1+3\nu)ax^2+\frac{2}{3}a^2(1-\nu)x-\cancel{3(3+\nu)xy^2}+$$
$$-\cancel{(3+\nu)ay^2}+\frac{8}{27}(1+\nu)a^3\Big]$$

ESEMPIO DI PIASTRA ELLITTICA



1) BORDO INCASTRATO

$$W\big|_\Gamma = 0 \qquad \frac{\partial W}{\partial n}\Big|_\Gamma = 0$$

2) Consideriamo l'equazione del bordo (ellisse)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\Downarrow$$

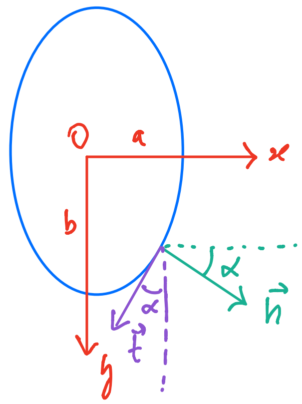
$$W = W_0\left(1-\frac{x^2}{a^2}-\frac{y^2}{b^2}\right)$$

superficie di prova

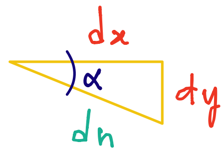
l'abbassamento nel punto $(x=0, y=0)$

$$\nabla_4 W = \frac{p_0}{D}$$

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p_0}{D}$$



PER USO FUTURO DELLE CONDIZIONI AL BORDO



$$dx = dn \cos\alpha \quad ; \quad dy = dn \sin\alpha$$

$$\cos\alpha = \frac{dx}{dn} \quad ; \quad \sin\alpha = \frac{dy}{dn}$$

$$\frac{\partial W}{\partial n} = \frac{\partial W}{\partial x}\frac{dx}{dn} + \frac{\partial W}{\partial y}\frac{dy}{dn}$$

$$W = W_0\left(1 + \frac{x^4}{a^4} + \frac{y^4}{b^4} - 2\frac{x^2}{a^2} - 2\frac{y^2}{b^2} + 2\frac{x^2 y^2}{a^2 b^2}\right)$$

$$\frac{\partial W}{\partial x} = W_0\left(\frac{4x^3}{a^4} - 4\frac{x}{a^2} + 4\frac{x y^2}{a^2 b^2}\right)$$

$$\frac{\partial^2 W}{\partial x^2} = W_0\left(\frac{12\, x^2}{a^4} - \frac{4}{a^2} + 4\frac{y^2}{a^2 b^2}\right)$$

$$\frac{\partial^3 W}{\partial x^3} = W_0\left(\frac{24 x}{a^4}\right) \quad ; \quad \frac{\partial^4 W}{\partial x^3} = \frac{W_0\, 24}{a^4}$$

$$\frac{\partial^4 w}{\partial y^4} = W_0 \frac{24}{b^4}$$

$$\frac{\partial^2 w}{\partial x^2} = W_0 \left(\frac{12 x^2}{a^4} - \frac{4}{a^2} + \frac{4 y^2}{a^2 b^2} \right)$$

$$\frac{\partial^3 w}{\partial x^2 \partial y} = W_0 \left(\frac{8 y}{a^2 b^2} \right) \quad ; \quad \frac{\partial^4 w}{\partial x^2 \partial y^2} = \frac{W_0\, 8}{a^2 b^2}$$

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{P_0}{D}$$

$$W_0 \left(\frac{24}{a^4} + \frac{16}{a^2 b^2} + \frac{24}{b^4} \right) = \frac{P_0}{D}$$

$$\boxed{W_0 = \frac{P_0}{8 D} \frac{a^4 b^4}{3a^4 + 2a^2 b^2 + 3b^4}}$$

ADESSO VERIFICHIAMO LE CONDIZIONI AL BORDO

1) $w\big|_{\Gamma} = 0$ è evidente

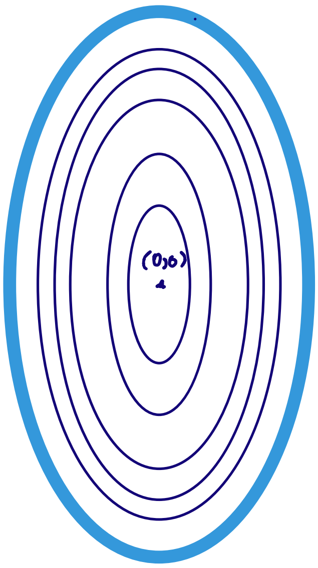
2) $\frac{\partial w}{\partial n}\Big|_{\Gamma} = 0$

$$\frac{\partial W}{\partial n} = \frac{\partial W}{\partial x}\cos\alpha + \frac{\partial W}{\partial y}\sin\alpha$$

$$= W_0\left(\frac{4x^3}{a^4} - \frac{4x}{a^2} + \frac{4xy^2}{a^2b^2}\right)\cos\alpha + W_0\left(\frac{4y^3}{b^4} - \frac{4y}{b^2} + \frac{4x^2y}{a^2b^2}\right)\sin\alpha$$

$$= W_0\,\frac{4x}{a^2}\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)\cos\alpha + W_0\,\frac{4y}{b^2}\left(\frac{y^2}{b^4} + \frac{x^2}{a^2} - 1\right)\sin\alpha$$

$$\left.\frac{\partial W}{\partial n}\right|_{\Gamma:\ \frac{x^2}{a^2}+\frac{y^2}{b^2}=1} = W_0\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)\left(\frac{4x}{a^2}\cos\alpha + \frac{4y}{b^2}\sin\alpha\right) = 0$$



$$W(0,0) = W_0 = W_{max}$$

$$= \frac{P_0}{8D}\,\frac{a^4b^4}{(3a^4 + 2a^2b^2 + 3b^4)}$$

VALUTARE M_x, M_y e M_{xy}: 1) al centro $(0,0)$

2) lungo i semiassi $a \rightarrow y=0$

$b \rightarrow x=0$

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right)$$

$$= -D W_0 \left[\frac{4}{a^2}\left(3\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right) + \nu \frac{4}{b^2}\left(3\frac{y^2}{b^2} + \frac{x^2}{a^2} - 1\right)\right]$$

$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right)$$

$$= -D W_0 \left[\frac{4}{b^2}\left(3\frac{y^2}{b^2} + \frac{x^2}{a^2} - 1\right) + \nu \frac{4}{a^2}\left(3\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)\right]$$

$$M_{xy} = -2 W_0 D \left(\frac{1-\nu}{2}\right)\left(\frac{\partial^2 w}{\partial x \partial y}\right) = -W_0 D (1-\nu) \frac{8xy}{a^2 b^2}$$

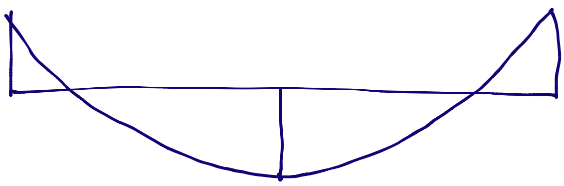
i $(x=0\,;\ y=0)$ $\qquad M_{xy}(0,0) = 0$

su $y=0$ e su $x=0$ (separatamente) si ottengono degli andamenti parabolici per M_x, M_y ($M_{xy} = 0$)

CALCOLARE per ESERCIZIO

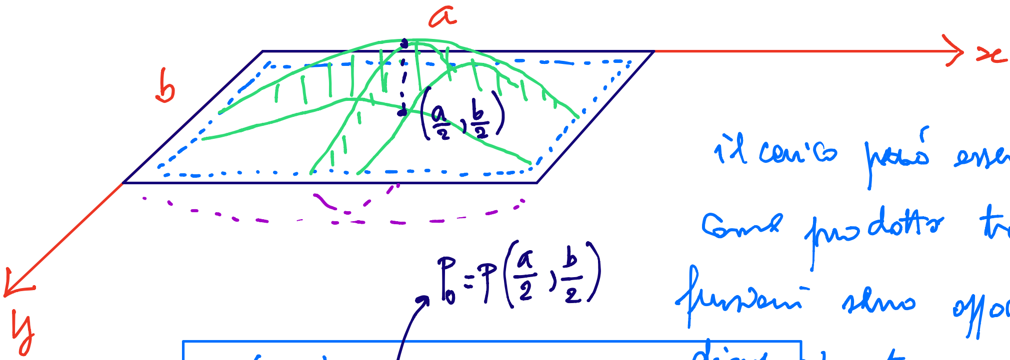
per esempio

M_x



PIASTRA RETTANGOLARE CON CARICO A "BOLLA": CARICO sinusoidale lungo x e y

appoggiata su tutti i lati



$p_0 = p\left(\frac{a}{2}, \frac{b}{2}\right)$

il carico può essere scritto come prodotto tra le due funzioni seno opportunamente dimensionate

$$\boxed{p(x,y) = p_0 \sin\frac{\pi x}{a} \cdot \sin\frac{\pi y}{b}}$$

superficie di prova

$$W(x,y) = W_0 \sin\frac{\pi x}{a} \sin\frac{\pi y}{b}$$

$$\nabla^4 W = \frac{p}{D} \qquad \text{cioè}$$

$$\frac{\partial^4 W}{\partial x^4} + 2\frac{\partial^4 W}{\partial x^2 \partial y^2} + \frac{\partial^4 W}{\partial y^4} = \frac{p_0}{D} \sin\frac{\pi x}{a} \sin\frac{\pi y}{b}$$

CONDIZIONI AL BORDO

su $x=0$ e $x=a$ $\qquad W=0$ e $M_x=0$

su $y=0$ e $y=b$ $\qquad W=0$ e $M_y=0$

$$\frac{d}{dx}\sin\frac{\pi x}{a} = \frac{\pi}{a}\cos\frac{\pi x}{a} \quad ; \quad \frac{d}{dx}\cos\frac{\pi x}{a} = -\frac{\pi}{a}\sin\frac{\pi x}{a}$$

$$\frac{d}{dy}\sin\frac{\pi y}{b} = \frac{\pi}{b}\cos\frac{\pi y}{b} \quad ; \quad \frac{d}{dy}\cos\frac{\pi y}{b} = -\frac{\pi}{b}\sin\frac{\pi y}{b}$$

$$\frac{\partial W}{\partial x} = W_0\,\frac{\pi}{a}\cos\frac{\pi x}{a}\sin\frac{\pi y}{b} \qquad \ldots\ldots$$