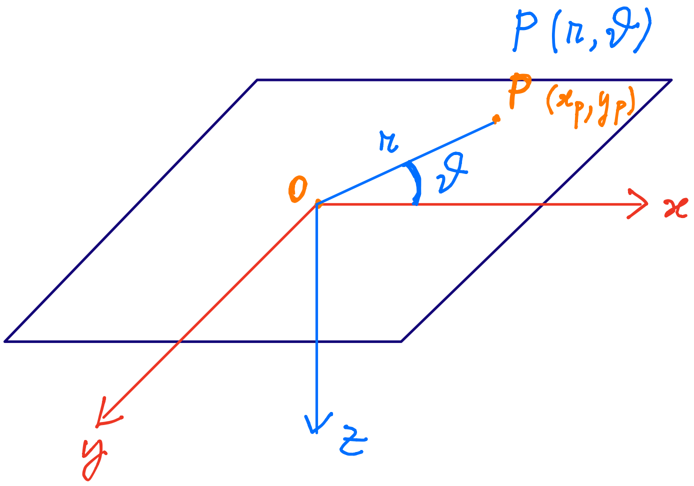
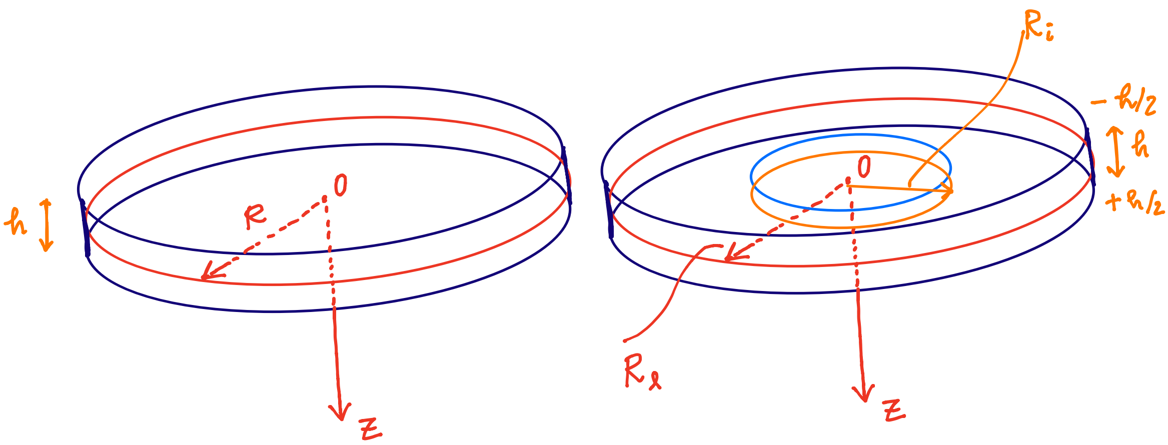
PIASTRE CIRCOLARI ASSIALSIMMETRICHE

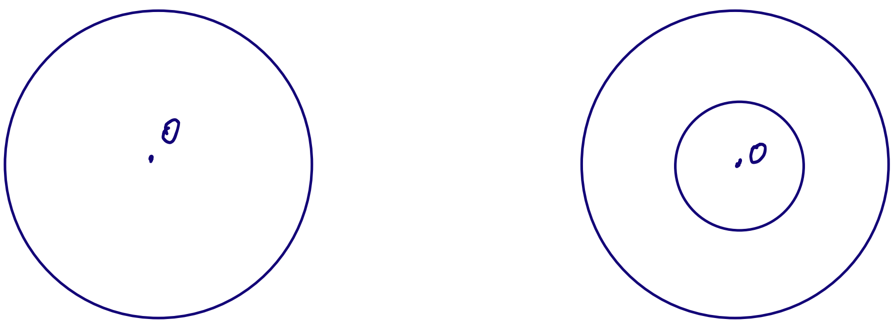


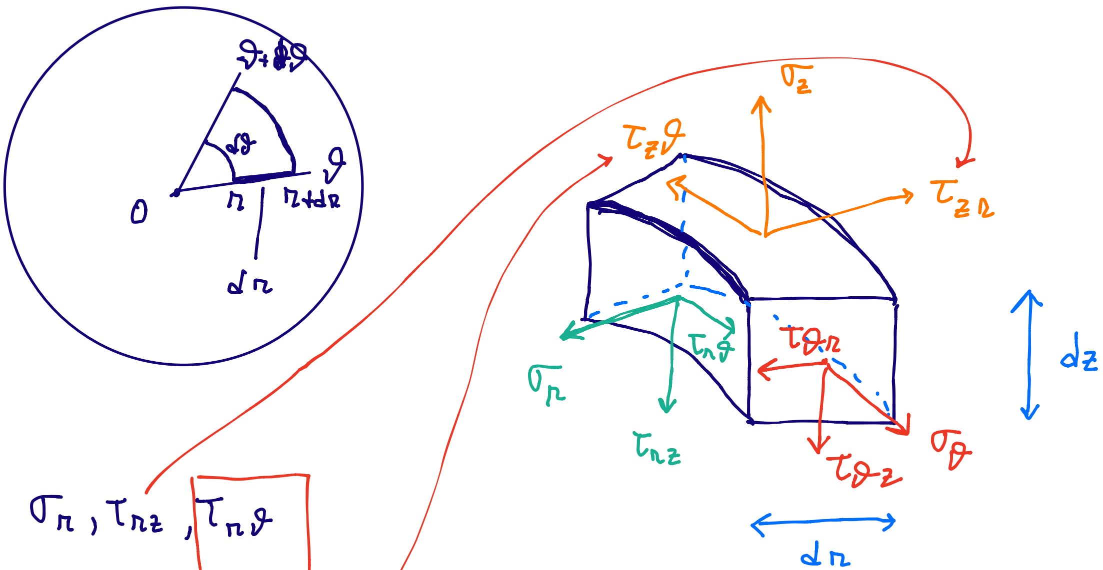
→ nello spazio le coordinate polari vengono sostituite dalle coordinate cilindriche

$$P(z, r, \vartheta)$$



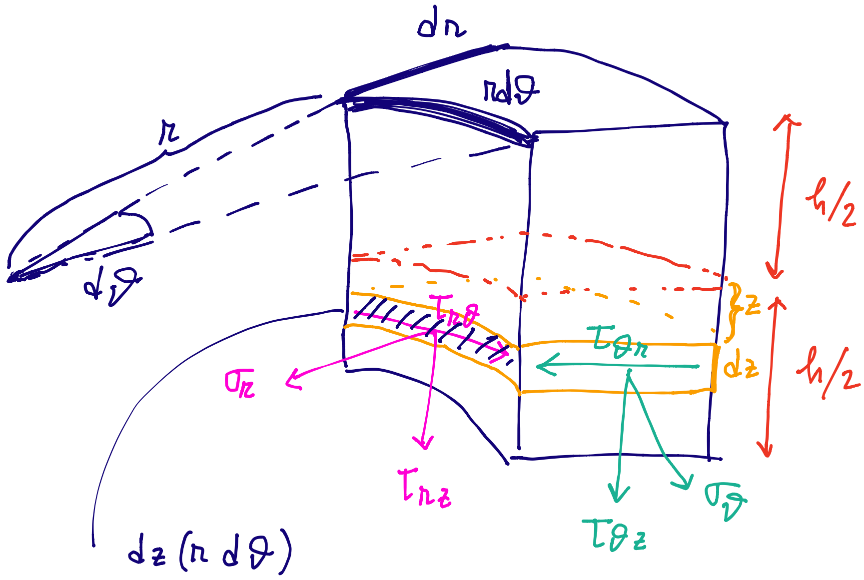
piastra circolare — corona circolare





$\sigma_r\,,\ \tau_{rz}\,,\ \boxed{\tau_{r\vartheta}}$

$\sigma_\vartheta\,,\ \tau_{\vartheta z}\,,\ \boxed{\tau_{\vartheta r}}$



$$N_r \cdot r\,d\vartheta = \int_{-h/2}^{h/2} \sigma_r \, dz \; r\,d\vartheta$$

$$N_r = \int_{-h/2}^{h/2} \sigma_r \, dz$$

$$N_{r\vartheta} \cdot r\,d\vartheta = \int_{-h/2}^{h/2} \tau_{r\vartheta}\,dz \cdot r\,d\vartheta \Rightarrow N_{r\vartheta} = \int_{-h/2}^{h/2} \tau_{r\vartheta}\,dz$$

$$T_{rz} \cdot r\,d\vartheta = \int_{-h/2}^{h/2} \tau_{rz}\,dz \cdot r\,d\vartheta \Rightarrow T_{rz} = \int_{-h/2}^{h/2} \tau_{rz}\,dz$$

sulle facce $dr\,dz$: $\sigma_\vartheta \;,\; \tau_{\vartheta r} \;,\; \tau_{\vartheta z}$

$$\begin{array}{ccc} \sigma_\vartheta & \tau_{\vartheta r} & \tau_{\vartheta z} \\ \downarrow & \downarrow & \downarrow \\ N_\vartheta & N_{\vartheta r} & T_{\vartheta z} \end{array}$$

$$N_\vartheta = \int_{-h/2}^{h/2} \sigma_\vartheta\,dz \;;\; N_{\vartheta r} = \int_{-h/2}^{h/2} \tau_{\vartheta r}\,dz \;;\; T_{\vartheta z} = \int_{-h/2}^{h/2} \tau_{\vartheta z}\,dz$$

MOMENTI : generati dalle componenti del tensore degli sforzi NON in direzione z (NO τ_{rz} , NO $\tau_{\vartheta z}$)

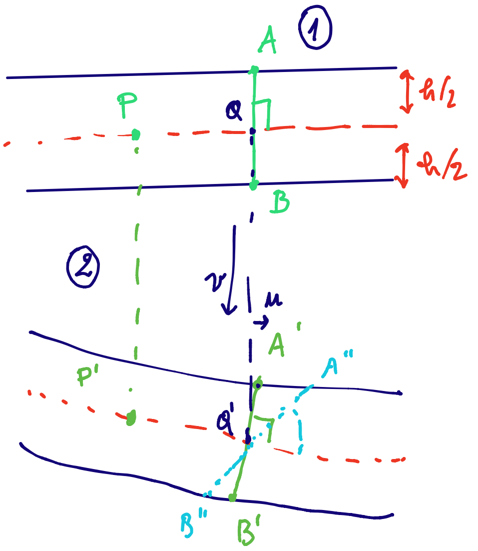
$$M_r = \int_{-h/2}^{h/2} \sigma_r\,z\,dz \;;\; M_\vartheta = \int_{-h/2}^{h/2} \sigma_\vartheta\,z\,dz \;;\; M_{r\vartheta} = \int_{-h/2}^{h/2} \tau_{r\vartheta}\,z\,dz$$

$$M_{\vartheta r} = \int_{-h/2}^{h/2} \tau_{\vartheta r}\,z\,dz$$

momenti flettenti

momenti torcenti

ADOTTIAMO ADESSO le ipotesi di piastra di Kirchhoff



$\overline{AB} = \overline{A'B'} \neq \overline{A''B''}$

①

$\Rightarrow \quad \gamma_{rz} = 0$

$\gamma_{\theta z} = 0$

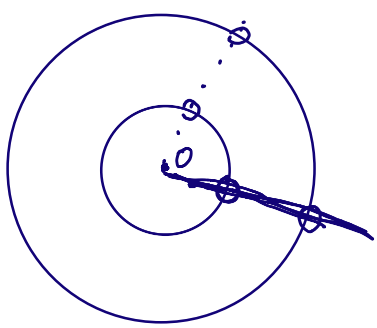
② i punti appartenenti alla superficie media della piastra subiscono solo uno spostamento ortogonale alla superficie

① e ② $\Rightarrow$ σ_r , σ_θ , $\tau_{r\theta} = \tau_{\theta r}$ sono lineari

$\Downarrow$

N_r , N_θ , $N_{r\theta}$ e $N_{\theta r}$ sono nulli

SIMMETRIA ASSIALE



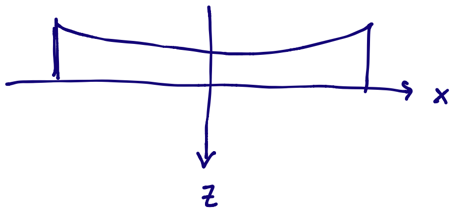
$\Rightarrow$ tutte le funzioni in gioco dipendono solo da r e non da ϑ

La richiesta che il problema dell'equilibrio rispetti una simmetria assiale impone dei vincoli sia sulle condizioni al bordo che sui carichi da considerare

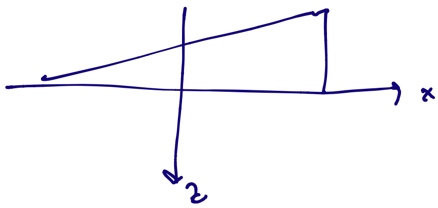
C B . 1) incastrato per intero
2) appoggiato per intero
3) libero per intero

sia per il bordo esterno che per l'eventuale bordo interno
TRA LORO INDIPENDENTI

CARICO



simmetrico

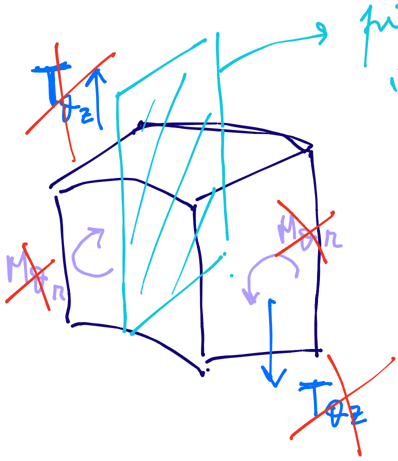


non è simmetrico

$\Downarrow$

NON lo considero

Se il problema è effettivamente assialsimmetrico:



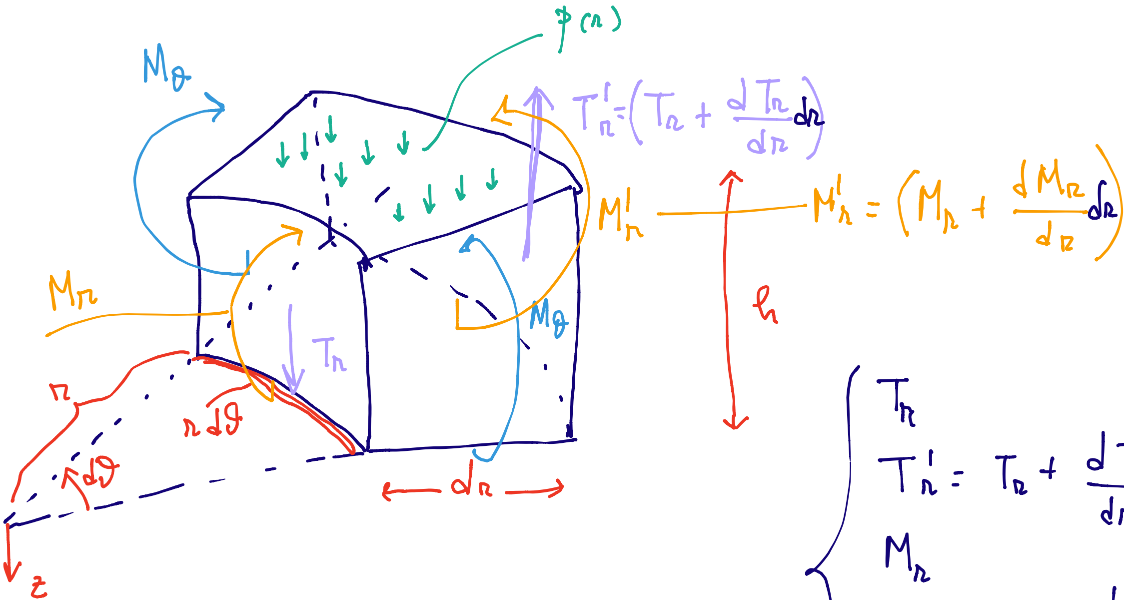
piano rispetto a cui il comportamento deve essere simmetrico

$$T_{\theta z} = 0$$

$$M_{\theta r} = M_{r\theta} = 0$$

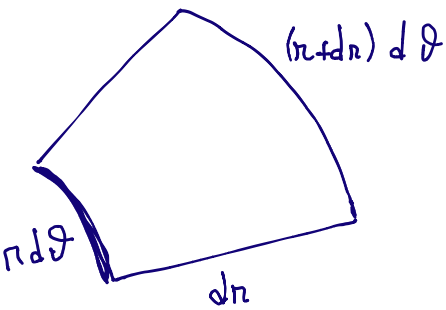
COSA RESTA?

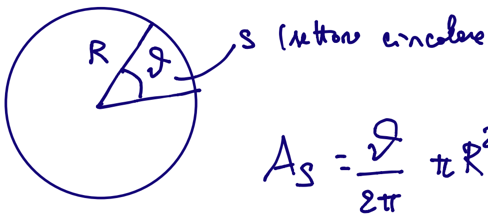
T_r , M_r , M_θ dipendono solo da r



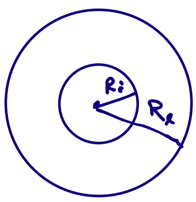
$$\begin{cases} T_r \\ T_r' = T_r + \dfrac{dT_r}{dr}\,dr \\ M_r \\ M_r' = M_r + \dfrac{dM_r}{dr}\,dr \\ M_\theta \\ p(r) \end{cases}$$

NOTA: qui sono derivate totali!





$$A_s = \frac{\theta}{2\pi}\,\pi R^2$$



$$A_c = \pi R_e^2 - \pi R_i^2$$

$$dS = \cancel{\pi}(r+dr)^2 \frac{d\vartheta}{2\cancel{\pi}} - \cancel{\pi} r^2 \frac{d\vartheta}{2\cancel{\pi}}$$

$$= (r^2 + 2r\,dr + dr^2)\frac{d\vartheta}{2} - r^2 \frac{d\vartheta}{2}$$

$$= \cancel{r^2 \frac{d\vartheta}{2}} + \cancel{2} r\,dr \frac{d\vartheta}{\cancel{2}} + (dr^2)\frac{d\vartheta}{2} - \cancel{r^2 \frac{d\vartheta}{2}}$$

$$= r\,dr\,d\vartheta$$

infinitesimo di ordine superiore

1) $R_z = 0$

$$T_r\, r\, d\vartheta - T'_r (r+dr)\, d\vartheta + p\, r\, dr\, d\vartheta = 0$$

$$T_r\, r\, d\vartheta - \left(T_r + \frac{dT_r}{dr} dr\right)(r+dr)\, d\vartheta + p\, r\, dr\, d\vartheta = 0$$

$$\cancel{T_r\, r\, d\vartheta} - \cancel{T_r\, r\, d\vartheta} - T_r\, dr\, d\vartheta - \frac{dT_r}{dr} r\, dr\, d\vartheta +$$

trascuriamo

$$- \frac{dT_r}{dr} (dr^2)\, d\vartheta + p\, r\, dr\, d\vartheta = 0$$

$$-T_r\, dr\, d\vartheta - \frac{dT_r}{dr}\, r\, dr\, d\vartheta + p\, r\, dr\, d\vartheta = 0$$

$$\left[\left(-\frac{T_r}{r} - \frac{dT_r}{dr}\right) + p\right] r\, dr\, d\vartheta = 0$$

$$\left[-\left(\frac{dT_r}{dr}+\frac{T_r}{r}\right)+p\right] r\,dr\,d\vartheta=0$$

$$\underbrace{r\frac{dT_r}{dr}+T_r}=pr$$

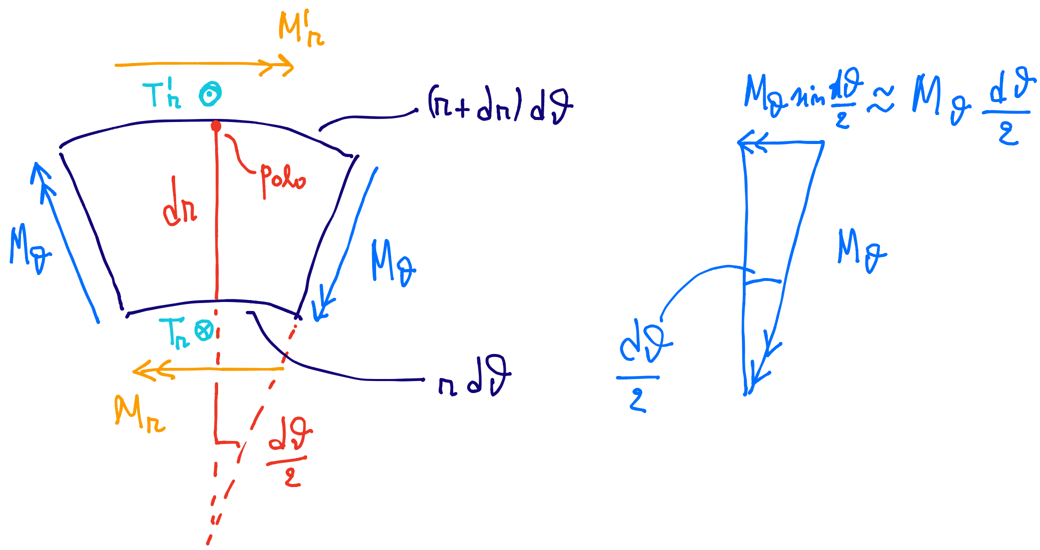
$$\frac{d}{dr}(rT_r)=T_r+r\frac{dT_r}{dr}$$

$$\Downarrow$$

$$\boxed{\frac{d}{dr}(rT_r)=pr}$$

prima equazione d'equilibrio

EQUILIBRIO ALLA ROTAZIONE



$$M_{\perp r} = 0$$

$$-M_\theta\, dr \frac{d\theta}{2} - M_r\, r\, d\theta - M_\theta\, dr \frac{d\theta}{2} + M_r' (r + dr)\, d\theta + T_r\, r\, d\theta\, dr = 0$$

$$-M_r\, r\, d\theta - \cancel{2}\, M_\theta\, dr \frac{d\theta}{\cancel{2}} + \left(M_r + \frac{dM_r}{dr} dr\right)(r + dr)\, d\theta + T_r\, r\, d\theta\, dr = 0$$

$$-\cancel{M_r\, r\, d\theta} - M_\theta\, dr\, d\theta + \cancel{M_r\, r\, d\theta} + M_r\, dr\, d\theta + \frac{dM_r}{dr}\, r\, dr\, d\theta + \frac{dM_r}{dr} \overbrace{dr^2}^{\text{trascurabile}} d\theta + T_r\, r\, dr\, d\theta = 0$$

$$(M_r - M_\theta)\, dr\, d\theta + r \frac{dM_r}{dr}\, dr\, d\theta + r\, T_r\, dr\, d\theta = 0$$

II Cardinale equazione

$$\boxed{M_r - M_\theta + r \frac{dM_r}{dr} + r\, T_r = 0}$$

Deriva questa equazione rispetto a r

$$\frac{d}{dr}(M_r - M_\theta) + \frac{d}{dr}\left(r \frac{dM_r}{dr}\right) + \frac{d}{dr}(r\, T_r) = 0$$

↓ tramite la prima equazione

$p\, r$

$$\boxed{\frac{d}{dr}\left(M_r - M_\theta\right) + \frac{d}{dr}\left(r\,\frac{dM_r}{dr}\right) + pr = 0}$$

seconda equazione