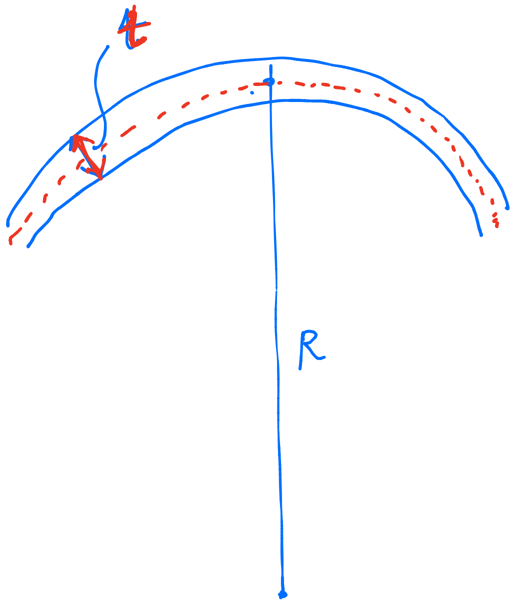
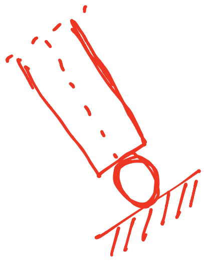
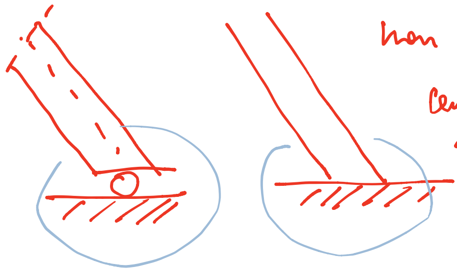
GUSCI – TEORIA DEI GUSCI SOTTILI



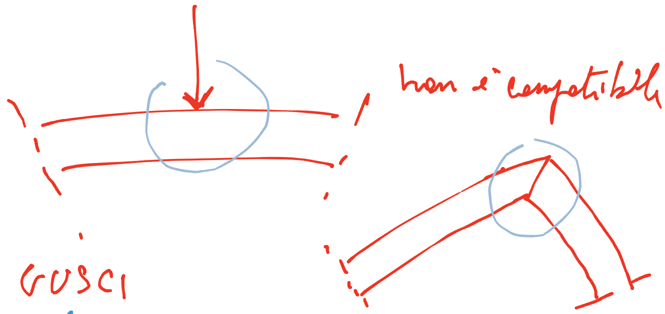
$$\frac{t}{R} \ll \frac{1}{50} \Rightarrow \text{guscio sottile}$$



Compatibile con la teoria membranale

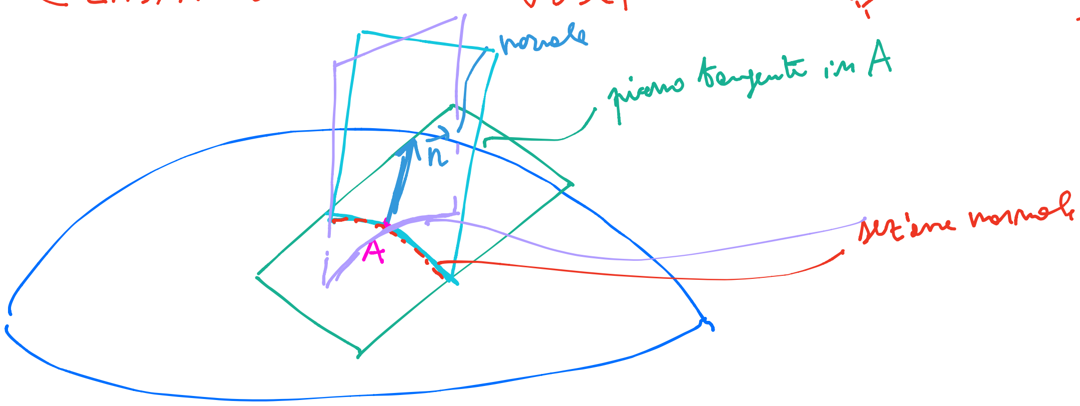


non compatibili con la teoria membranale



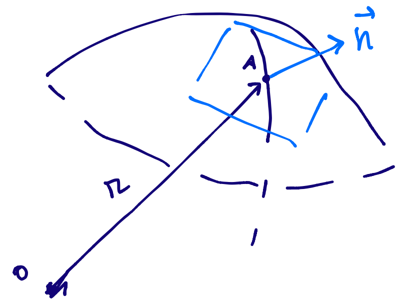
non è compatibile

CLASSIFICAZIONE DI GUSCI



A ciascuna sezione normale corrisponde una curvatura locale k e un relativo raggio di curvatura r

$$k = \pm \frac{1}{r}$$



se l'origine del raggio di curvatura è positiva rispetto a $\vec{n}$ allora $k = \frac{1}{r}$

se " " " negativa rispetto a $\vec{n}$ allora $k = -\frac{1}{r}$

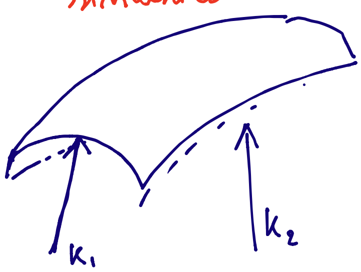
Ci saranno (ovviamente) un valore minimo e un valore massimo di k in A

k_1 e k_2 curvature principali

le due sezioni principali (corrispondenti a k_1 e k_2) sono ortogonali

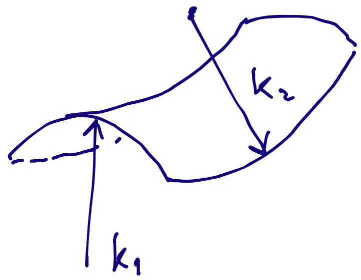
CURVATURA GAUSSIANA $\quad k_g = k_1 \cdot k_2$

sinclastico



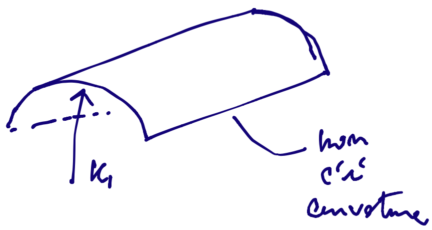
$k_g = k_1 \cdot k_2 > 0$

anticlastico



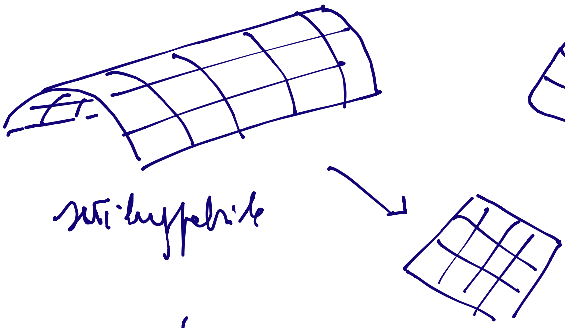
$k_g = k_1 \cdot k_2 < 0$

a singola curvatura



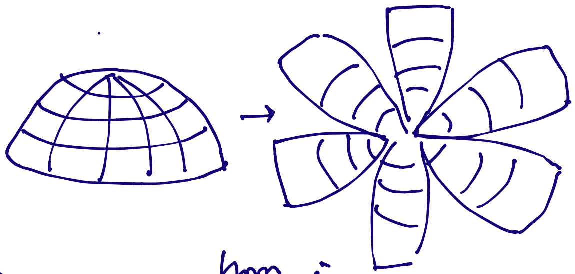
$k_g = 0$

Superfici sviluppabili e non-sviluppabili



sviluppabile

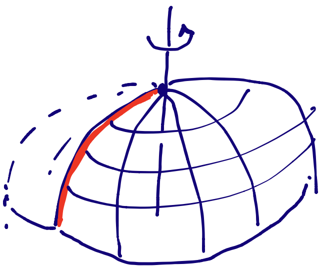
$k_g = 0$

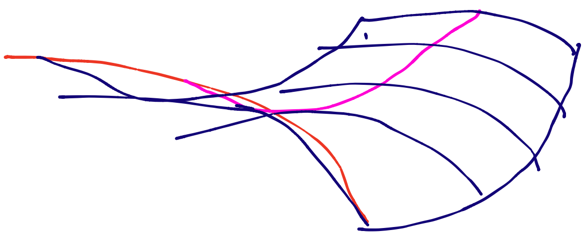


non è sviluppabile

$k_g \neq 0$

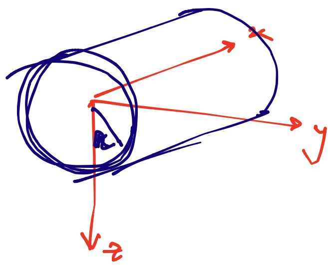
Superfici di rivoluzione, superfici di traslazione





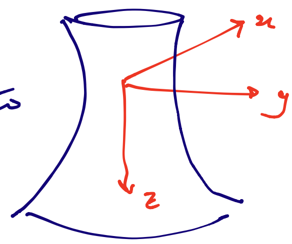
DESCRIZIONE ANALITICA DEI GUSCI (DELLA SUPERFICIE MEDIA)

$x^2 + y^2 = r^2$ cilindro

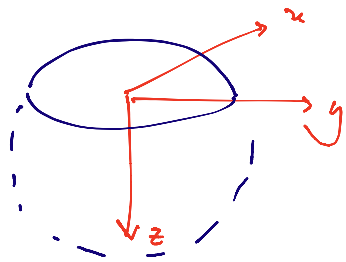


$$\frac{x^2+y^2}{a^2} - \frac{z^2}{b^2} = 1$$

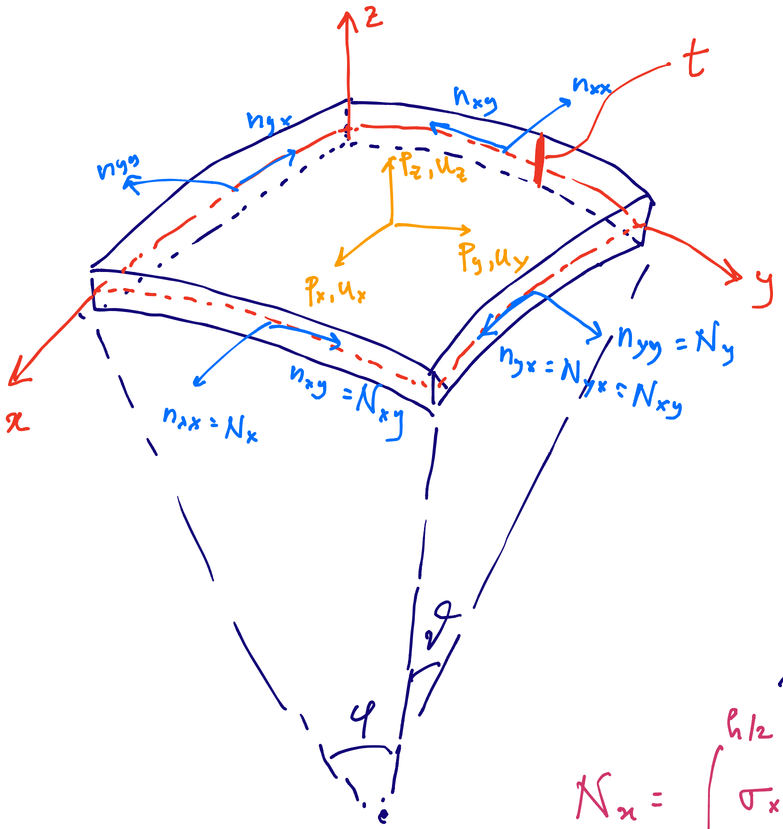
torre di raffreddamento



$$\frac{x^2+y^2+z^2}{a^2} = 1$$



TEORIA MEMBRANALE PER GUSCI CON CURVATURE PRINCIPALI



$k_1 < 0$ e $k_2 < 0$

$$\vec{u} = (u_x, u_y, u_z)^t$$

$$\vec{p} = (p_x, p_y, p_z)^t$$

$$\vec{e} = (\varepsilon_x, \varepsilon_y, \gamma_{xy})^t$$

$$\vec{\sigma} = (\sigma_x, \sigma_y, \tau_{xy})^t$$

$$\Downarrow$$

$$\vec{S} = (N_x, N_y, N_{xy})^t$$

sul libro $\quad n_{xx} \quad n_{yy} \quad n_{xy}$

$$N_x = \int_{-h/2}^{h/2} \sigma_x \, dz$$ per la piastra

ADESSO DOVREMMO TENERE CONTO DELLA CURVATURA

$$N_x = \int_{-t/2}^{t/2} \sigma_x \left(1 - \left(\frac{z}{r}\right)\right) dz \simeq \int_{-t/2}^{t/2} \sigma_x \, dz$$ TEORIA MEMBRANALE

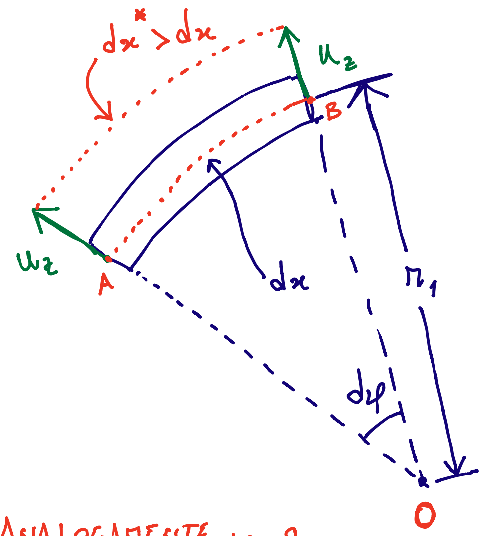
per gusci sottili è trascurabile

$$\begin{pmatrix} u_x \\ u_y \end{pmatrix} \quad \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} \quad \begin{pmatrix} n_{xx} \\ n_{yy} \\ n_{xy} \end{pmatrix} \quad \begin{pmatrix} p_x \\ p_y \end{pmatrix}$$

cinematica — legame costitutivo — equazioni di equilibrio

CINEMATICA

$$\left[\varepsilon_x = \frac{\partial u_x}{\partial x} \; ; \; \varepsilon_y = \frac{\partial u_y}{\partial y} \; ; \; \gamma_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right]$$ per una piastra



$$dx = r_1 \sin(d\varphi) \approx r_1 \, d\varphi$$

$$\varepsilon_x = \frac{dx^* - dx}{dx}$$

$$= \frac{(r_1 + u_z)\, d\varphi - r_1 \, d\varphi}{r_1 \, d\varphi}$$

$$= \frac{u_z}{r_1} = \boxed{-k_1 u_z = \varepsilon_x}$$

ANALOGAMENTE per ε_y

$$\boxed{\varepsilon_y = \frac{u_z}{r_2} = -k_2 u_z}$$

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} \partial/\partial x & 0 & -k_1 \\ 0 & \partial/\partial y & -k_2 \\ \partial/\partial y & \partial/\partial x & 0 \end{pmatrix} \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} \quad , \quad \vec{e} = \underline{B}\,\vec{u}$$

operatore differenziale

LEGAME COSTITUTIVO

Consideriamo che il materiale del guscio sia elastico lineare e quindi sia governato da una legge alla Hooke

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{pmatrix} = \frac{E}{1-\nu^2} \begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{pmatrix} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix}$$

$$\begin{pmatrix} n_{xx} \\ n_{yy} \\ n_{xy} \end{pmatrix} = \frac{E\,t}{1-\nu^2} \begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{pmatrix} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix}$$

$$n_{ij} = \int_{-t/2}^{t/2} \sigma_{ij} \left(1 - \frac{z}{R}\right) dz \overset{\text{per gusci sottili}}{\approx} \int_{-t/2}^{t/2} \sigma_{ij}\, dz \overset{\text{isotropia e spessori costanti}}{=} t\,\sigma_{ij}$$

$$\underline{D} = \frac{Et}{1-\nu^2} \begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{pmatrix}$$

MATRICE DI RIGIDEZZA

$$\vec{S} = \underline{D}\,\vec{e}$$

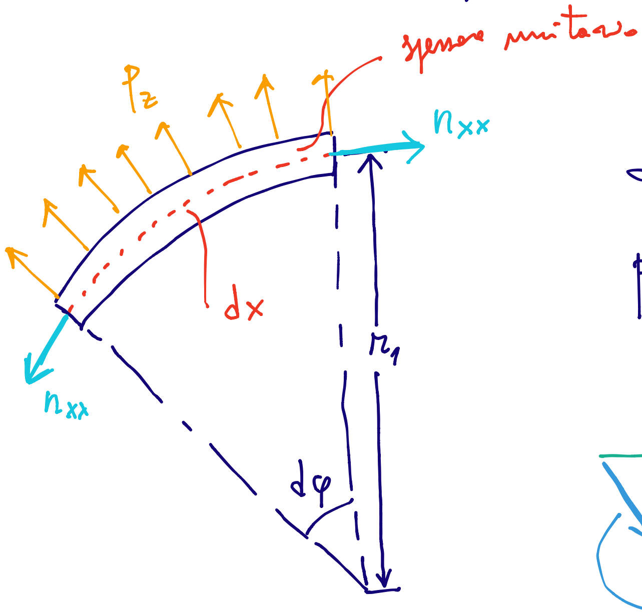
EQUAZIONI D'EQUILIBRIO

$$\begin{cases} \dfrac{\partial n_{xx}}{\partial x} + \dfrac{\partial n_{xy}}{\partial y} + p_x = 0 \\[2ex] \dfrac{\partial n_{xy}}{\partial x} + \dfrac{\partial n_{yy}}{\partial y} + p_y = 0 \end{cases}$$

+ un'altra espressione che tenga conto di u_z e p_z

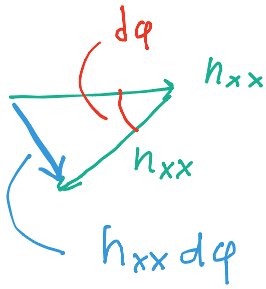
$\Downarrow$

k_1 e k_2



$$dx = r_1\, d\varphi$$

$$p_z\, dx = p_z\, r_1\, d\varphi$$

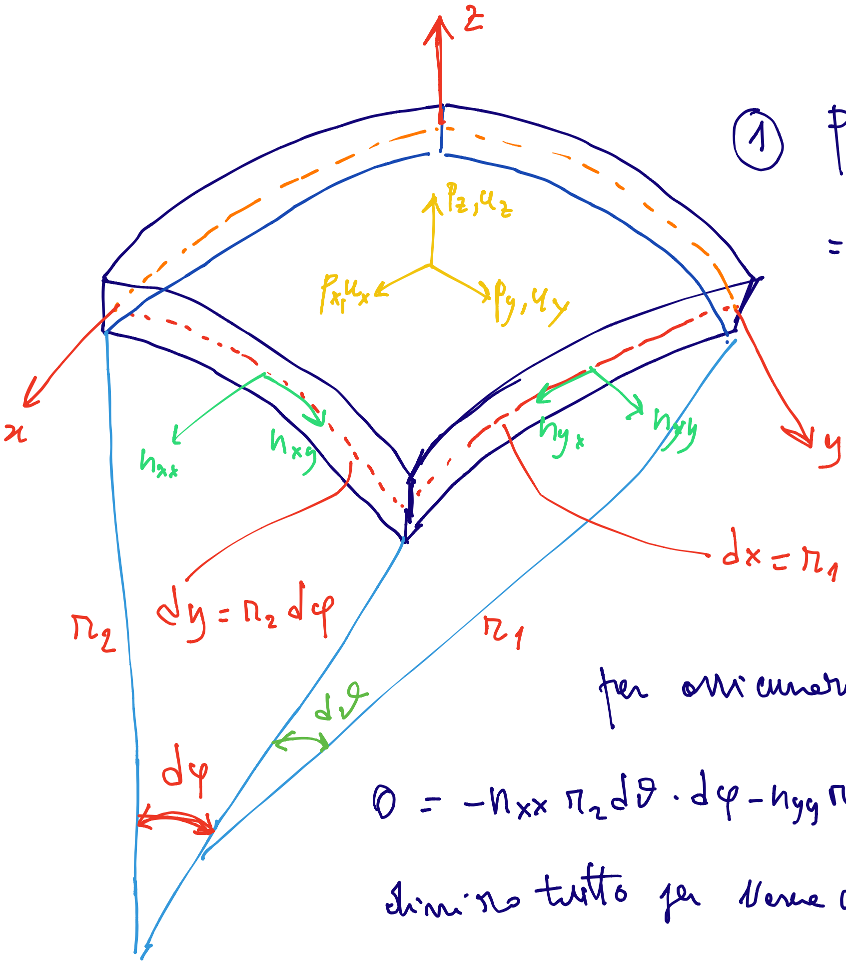


$$-n_{xx}\, d\varphi + p_z\, r_1\, d\varphi = 0$$

dividiamo tutto per $dx = r_1 d\varphi$

$$-\frac{n_{xx}}{r_1} + p_z = 0 \Rightarrow k_1\, n_{xx} + p_z = 0$$

$$k_2\, n_{yy} + p_z = 0$$



1 $p_z\, dx\, dy =$

$= p_z\, r_1\, d\vartheta\, r_2\, d\varphi$

2 $-n_{xx}\, r_1\, d\vartheta$

3 $-n_{yy}\, r_2\, d\varphi$

per ottenere l'equilibrio

$$0 = -n_{xx}\, r_2\, d\vartheta \cdot d\varphi - n_{yy}\, r_1\, d\varphi \cdot d\vartheta + p_z\, r_1\, d\varphi\, r_2\, d\vartheta$$

divido tutto per l'area $r_1\, d\varphi \cdot r_2\, d\vartheta$

$$-\frac{n_{xx}}{r_1} - \frac{n_{yy}}{r_2} + p_z = 0$$

$$\boxed{k_1 n_{xx} + k_2 n_{yy} + p_z = 0}$$

$$\begin{cases} \dfrac{\partial n_{xx}}{\partial x} + \dfrac{\partial n_{xy}}{\partial y} + p_x = 0 \\ \dfrac{\partial n_{xy}}{\partial x} + \dfrac{\partial n_{yy}}{\partial y} + p_y = 0 \\ k_1 n_{xx} + k_2 n_{yy} + p_z = 0 \end{cases}$$

in forma matriciale

$$\underline{B}^* \vec{S} = \vec{p}$$

$$\underline{B}^* = \begin{pmatrix} -\partial/\partial x & 0 & -\partial/\partial y \\ 0 & -\partial/\partial y & -\partial/\partial x \\ -k_1 & -k_2 & 0 \end{pmatrix}$$