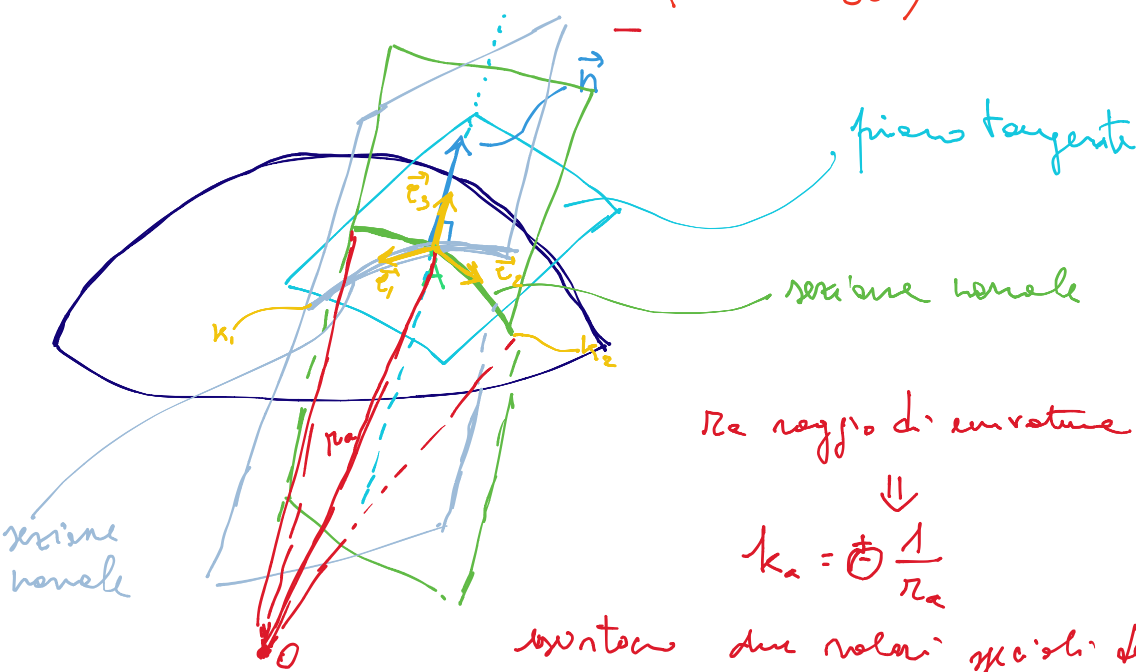
RIEPILOGO GUSCI SOTTILI $\left(\frac{t}{R} \ll \frac{1}{50}\right)$



R_α raggio di curvatura

$\Downarrow$

$$k_\alpha = \pm \frac{1}{R_\alpha}$$

assunto due valori speciali della curvatura: MAX e min

k_1 e k_2 curvature principali

$\Downarrow$

$\vec{e}_1, \vec{e}_2, \vec{e}_3 // \vec{n} \Leftarrow k_1 \perp k_2$ sezioni principali

la volta scorsa abbiamo considerato il sistema (x, y, z) con assi orientati lungo $\vec{e}_1, \vec{e}_2, \vec{e}_3$.

EQUAZIONI D'EQUILIBRIO

$$\begin{cases} \dfrac{\partial n_{xx}}{\partial x} + \dfrac{\partial n_{xy}}{\partial y} + p_x = 0 \\[2ex] \dfrac{\partial n_{xy}}{\partial x} + \dfrac{\partial n_{yy}}{\partial y} + p_y = 0 \\[2ex] k_1 n_{xx} + k_2 n_{yy} + p_z = 0 \end{cases}$$

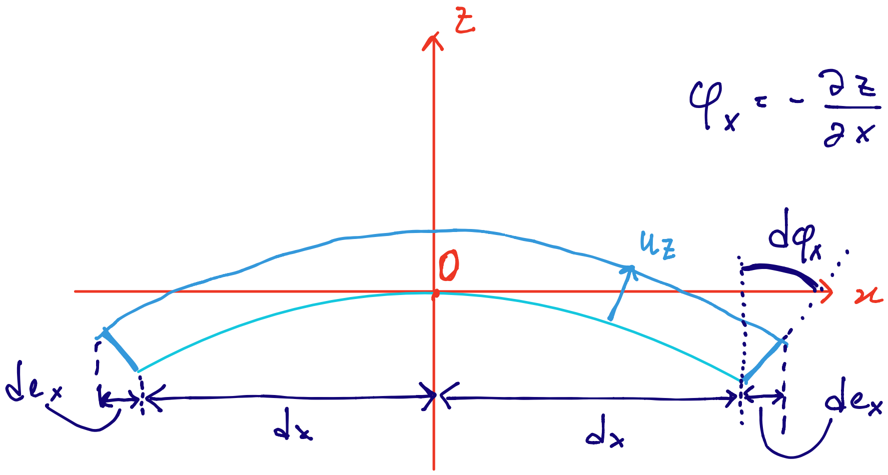
TEORIA MEMBRANALE PER GUSCI SOTTILI CON CURVATURE ARBITRARIE

$k_1, k_2 \longrightarrow k_x \,,\ k_y \,,\ k_{xy}$

• CINEMATICA

$$\varepsilon_x = \frac{\partial u_x}{\partial x} \,, \quad \varepsilon_y = \frac{\partial u_y}{\partial y} \,, \quad \gamma_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}$$

$$\varphi_x = -\frac{\partial z}{\partial x}$$



$$dx \longrightarrow \underline{dx + de_x = dx^*}$$

$$d\varphi_x = -\frac{\partial^2 z}{\partial x^2}\, dx \qquad de_x = u_z\, d\varphi_x = -\frac{\partial^2 z}{\partial x^2}\, dx\, u_z$$

$$\varepsilon_x = \frac{dx^* - dx}{dx} = \frac{de_x}{dx} = -\frac{\partial^2 z}{\partial x^2}\, u_z$$

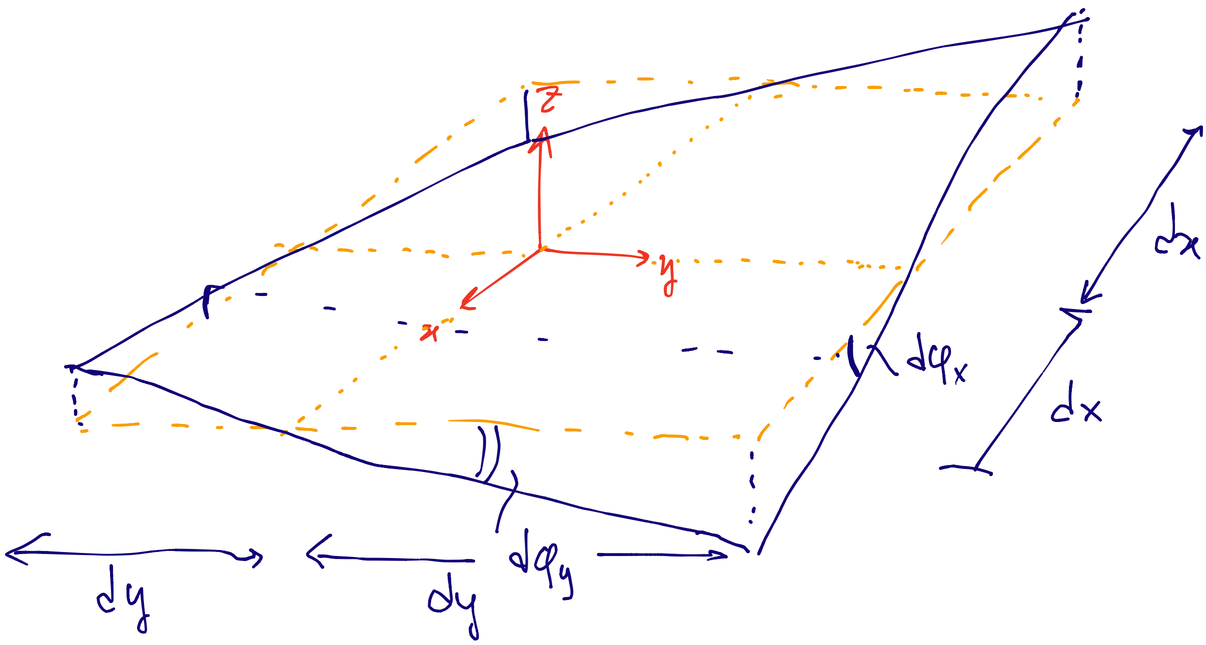
$$\Downarrow$$

$$k_x = \frac{\partial^2 z}{\partial x^2} \implies \boxed{\varepsilon_x = -k_x\, u_z}$$

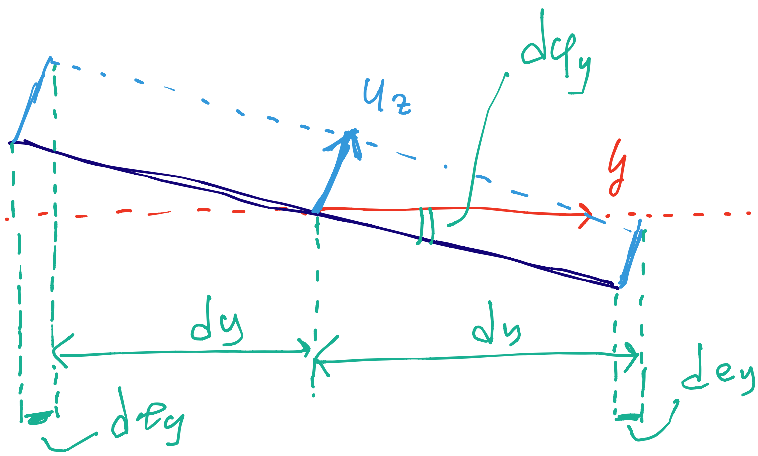
IDENTICAMENTE SI trova che

$$\boxed{\varepsilon_y = -k_y u_z} \quad \text{dove } k_y = \frac{\partial^2 z}{\partial y^2}$$

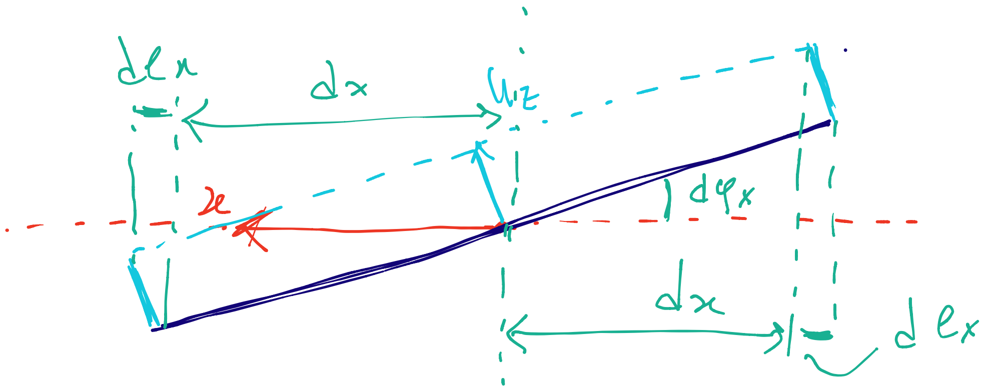
- PARTE DI TWIST k_{xy}



$$\varphi_y = -\frac{\partial z}{\partial y}$$



$$\varphi_x = -\frac{\partial z}{\partial x}$$



$$d\varphi_x = -\frac{\partial^2 z}{\partial y \, \partial x} \, dy \, , \qquad d\varphi_y = -\frac{\partial^2 z}{\partial x \, \partial y} \, dx$$

TEOREMA DI SCHWARZ

$$f : \Omega \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R} \quad \& \quad f \in C^2(\Omega)$$

ALLORA

$$\frac{\partial^2 f}{\partial x \, \partial y} = \frac{\partial^2 f}{\partial y \, \partial x}$$

$$d e_x = d\varphi_x \, u_z \qquad , \quad d e_y = d\varphi_y \, u_z$$

$$\gamma_{xy} = \varepsilon_{xy} + \varepsilon_{yx}$$

$$\boxed{\gamma_{xy} = -2 \, k_{xy} \, u_z} \qquad \text{dove} \quad k_{xy} = \frac{\partial^2 z}{\partial x \, \partial y}$$

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = \underbrace{\begin{pmatrix} \partial/\partial x & 0 & -k_x \\ 0 & \partial/\partial y & -k_y \\ \partial/\partial y & \partial/\partial x & -2k_{xy} \end{pmatrix}}_{\underline{B}} \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}$$

$$k_x = \frac{\partial^2 z}{\partial x^2} \quad , \quad k_y = \frac{\partial^2 z}{\partial y^2} \quad , \quad k_{xy} = \frac{\partial^2 z}{\partial x \partial y}$$

- LEGAME ELASTICO

$$\begin{pmatrix} n_{xx} \\ n_{yy} \\ n_{xy} \end{pmatrix} = \frac{Et}{1-\nu^2} \underbrace{\begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{pmatrix}}_{\underline{D}\ \text{matrice di rigidezza}} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix}$$

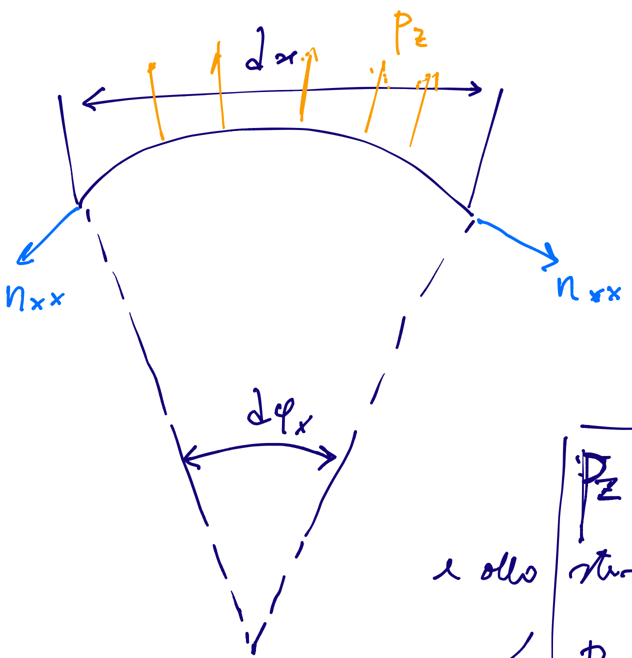
$$\vec{s} = \underline{D}\,\vec{e}$$

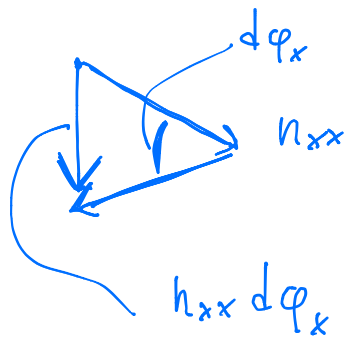
- EQUAZIONI D'EQUILIBRIO

$$\frac{\partial n_{xx}}{\partial x} + \frac{\partial n_{xy}}{\partial y} + p_x = 0$$

$$\frac{\partial n_{xy}}{\partial x} + \frac{\partial n_{yy}}{\partial y} + p_y = 0$$

EFFETTO DELLE CURVATURE





$$p_z\,dx - n_{xx}\,d\varphi_x = 0$$

e allo stesso modo ragiono per ottenere

$$p_z\,dy - n_{yy}\,d\varphi_y = 0$$

$$k_x\, n_{xx} + p_z = 0$$

$$k_y\, n_{yy} + p_z = 0$$

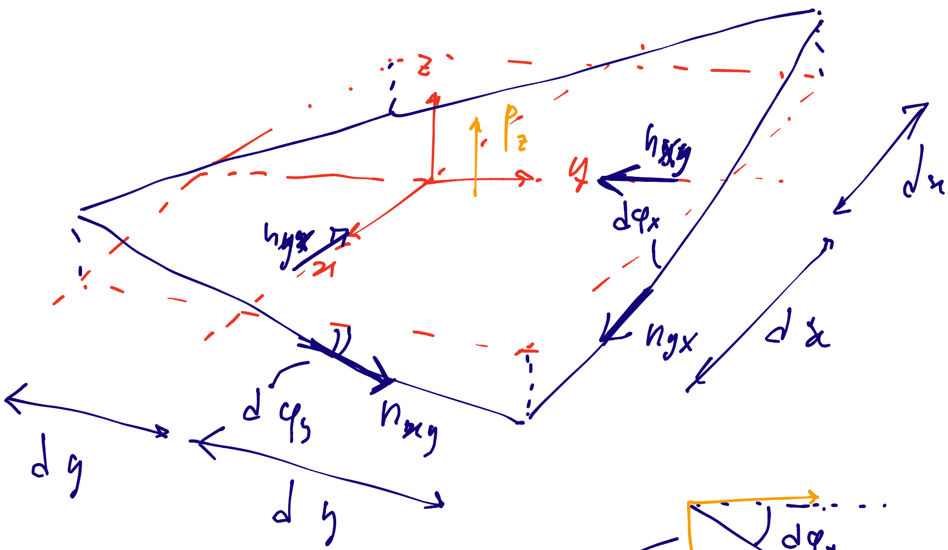
$$p_z\,dx\,dy - n_{xx}\,d\varphi_x\,dy - n_{yy}\,d\varphi_y\,dx = 0$$

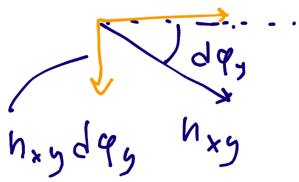
dividendo tutto per la superficie infinitesima

$$p_z - n_{xx}\frac{d\varphi_x}{dx} - n_{yy}\frac{d\varphi_y}{dy} = 0$$

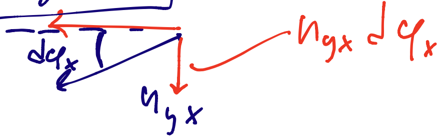
$$k_x n_{xx} + k_y n_{yy} + p_z = 0$$

EFFETTO DEL TWIST





$$\boxed{p_z\,dx\,dy - (n_{xy}\,d\varphi_y)\,dx - (n_{yx}\,d\varphi_x)\,dy = 0}$$



divido tutto per la superficie infinitesima $dx\,dy$

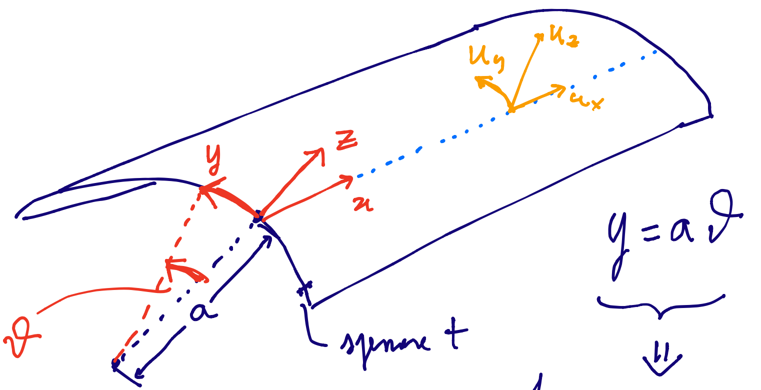
$$p_z - n_{xy}\frac{d\varphi_y}{dx} - n_{yx}\frac{d\varphi_x}{dy} = 0$$

$$2k_{xy}\, n_{xy} + p_z = 0$$

IN DEFINITIVA

$$\begin{pmatrix} -\partial/\partial x & 0 & -\dfrac{\partial}{\partial x} \\ 0 & -\partial/\partial y & -\partial/\partial x \\ -k_x & -k_y & -2k_{xy} \end{pmatrix} \begin{pmatrix} n_{xx} \\ n_{yy} \\ n_{xy} \end{pmatrix} = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}$$

GUSCI CILINDRICI CIRCOLARI (SOTTILI)



$$\underbrace{y = a\vartheta}$$

$$\Downarrow$$

$$k_y,\ p_y,\ u_y$$

$$\downarrow$$

$$n_{yy},\ n_{xy} \qquad \longleftarrow \qquad k_\vartheta,\ p_\vartheta,\ u_\vartheta$$

$$\downarrow$$

$$n_{\vartheta\vartheta},\ n_{x\vartheta}$$

$$(dx,\ \underbrace{a\,d\vartheta}_{dy})$$

la curvatura principale in direzione y è k_ϑ

$$\vec{u} = (u_x, u_\vartheta, u_z)^t \; ; \; \vec{e} = (\varepsilon_x, \varepsilon_\vartheta, \gamma_{x\vartheta})^t$$

$$\vec{s} = (n_{xx}, n_{\vartheta\vartheta}, n_{x\vartheta})^t \; ; \; \vec{q} = (p_x, p_\vartheta, p_z)$$

CINEMATICA

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_\vartheta \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} \partial/\partial x & 0 & 0 \\ 0 & \frac{1}{a}\frac{\partial}{\partial \vartheta} & 0 \\ \frac{1}{a}\frac{\partial}{\partial \vartheta} & \frac{\partial}{\partial x} & 0 \end{pmatrix} \begin{pmatrix} u_x \\ u_\vartheta \\ u_z \end{pmatrix}$$

LEGAME COSTITUTIVO

$$\begin{pmatrix} n_{xx} \\ n_{\vartheta\vartheta} \\ n_{x\vartheta} \end{pmatrix} = \frac{Et}{1-\nu^2} \begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{pmatrix} \begin{pmatrix} \varepsilon_x \\ \varepsilon_\vartheta \\ \gamma_{x\vartheta} \end{pmatrix}$$

EQUAZIONI D'EQUILIBRIO

$$\begin{pmatrix} -\frac{\partial}{\partial x} & 0 & -\frac{1}{a}\frac{\partial}{\partial \vartheta} \\ 0 & -\frac{1}{a}\frac{\partial}{\partial \vartheta} & -\frac{\partial}{\partial x} \\ 0 & \frac{1}{a} & 0 \end{pmatrix} \begin{pmatrix} n_{xx} \\ n_{\vartheta\vartheta} \\ n_{x\vartheta} \end{pmatrix} = \begin{pmatrix} p_x \\ p_\vartheta \\ p_z \end{pmatrix}$$

$$n_{\vartheta\vartheta} = a\, p_z$$

$$\underbrace{-\frac{1}{a}\frac{\partial}{\partial\vartheta} n_{\vartheta\vartheta}} - \frac{\partial}{\partial x} n_{x\vartheta} = p_\vartheta$$

$$-\frac{\partial}{\partial\vartheta} p_z - \frac{\partial}{\partial x} n_{x\vartheta} = p_\vartheta$$

$$\frac{\partial}{\partial x} n_{x\vartheta} = -\left(p_\vartheta + \frac{1}{a}\frac{\partial}{\partial\vartheta} n_{\vartheta\vartheta}\right)$$

$$\left.\begin{aligned} n_{x\vartheta} &= -\int\left(p_\vartheta + \frac{1}{a}\frac{\partial n_{\vartheta\vartheta}}{\partial\vartheta}\right)dx \\ n_{xx} &= -\int\left(p_x + \frac{1}{a}\frac{\partial n_{x\vartheta}}{\partial\vartheta}\right)dx \end{aligned}\right\} \Rightarrow 2 \text{ costanti di integrazione}$$