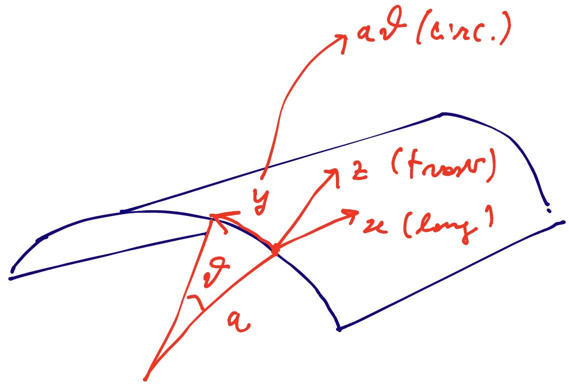
# GUSCI CILINDRICI

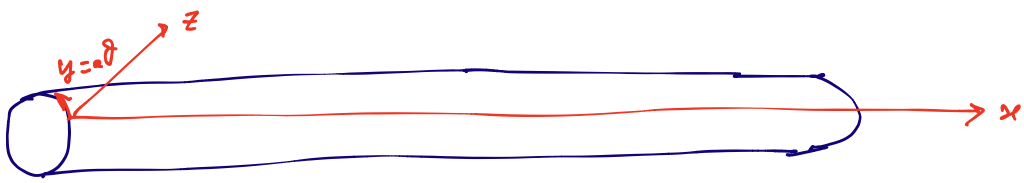
PER LE FORZE MEMBRANALI

$$\begin{cases} n_{\vartheta\vartheta} = a\, p_z \\ n_{x\vartheta} = -\displaystyle\int \left( p_\vartheta + \frac{1}{a}\frac{\partial n_{\vartheta\vartheta}}{\partial \vartheta} \right) dx \\ n_{xx} = -\displaystyle\int \left( p_x + \frac{1}{a}\frac{\partial n_{x\vartheta}}{\partial \vartheta} \right) dx \end{cases}$$




$$\begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{\vartheta\vartheta} \\ \gamma_{x\vartheta} \end{pmatrix} = \frac{1}{Et} \begin{pmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 2(1+\nu) \end{pmatrix} \begin{pmatrix} n_{xx} \\ n_{\vartheta\vartheta} \\ n_{x\vartheta} \end{pmatrix}$$

$$\begin{cases} \varepsilon_{xx} = \dfrac{1}{Et}\left(n_{xx} - \nu\, n_{\vartheta\vartheta}\right) \\ \varepsilon_{\vartheta\vartheta} = \dfrac{1}{Et}\left(n_{\vartheta\vartheta} - \nu\, n_{xx}\right) \\ \gamma_{x\vartheta} = \dfrac{1}{Et}\, 2(1+\nu)\, n_{x\vartheta} \end{cases} \rightarrow \begin{cases} u_x = \displaystyle\int \varepsilon_{xx}\, dx \\ u_\vartheta = \displaystyle\int \left( \gamma_{x\vartheta} - \frac{1}{a}\frac{\partial u_x}{\partial \vartheta} \right) dx \\ u_z = a\left( \varepsilon_{\vartheta\vartheta} - \dfrac{1}{a}\dfrac{\partial u_\vartheta}{\partial \vartheta} \right) \end{cases}$$




Se $p_x = 0$ e $p_z$, $p_\vartheta$ indipendenti da $x$, allora il problema si semplifica

$$n_{\vartheta\vartheta} = a\, p_z$$

$$n_{x\vartheta} = -\left( \int p_\vartheta \, dx + \int \frac{1}{a} \frac{\partial n_{\vartheta\vartheta}}{\partial \vartheta} \, dx \right)$$

$$= -\left( p_\vartheta \, x + \int \frac{\partial p_z}{\partial \vartheta} \, dx \right)$$

$$= -\left( p_\vartheta + \frac{\partial p_z}{\partial \vartheta} \right) x + f_1(\vartheta)$$

$$n_{xx} = -\int \left( \cancel{p_x} + \frac{1}{a} \frac{\partial n_{x\vartheta}}{\partial \vartheta} \right) dx$$

$$= \int \frac{1}{a} \frac{\partial}{\partial \vartheta} \left[ \left( p_\vartheta + \frac{\partial p_z}{\partial \vartheta} \right) x - f_1(\vartheta) \right] dx$$

$$= \frac{1}{2a} \left( \frac{\partial p_\vartheta}{\partial \vartheta} + \frac{\partial^2 p_z}{\partial \vartheta^2} \right) x^2 - \frac{1}{a} \frac{\partial f_1(\vartheta)}{\partial \vartheta} x + f_2(\vartheta)$$

SCEGLIAMO

$$p_x = 0$$

$$p_\vartheta = \hat{p}_\vartheta \sin\vartheta$$

$$p_z = \hat{p}_z \cos\vartheta$$

Con questo carico si ottiene

$$n_{\vartheta\vartheta} = a\, \hat{p}_z \cos\vartheta$$

$$n_{x\vartheta} = -\left(p_\vartheta + \frac{\partial p_z}{\partial \vartheta}\right) x + f_1(\vartheta)$$

$$= -\left[\hat{p}_\vartheta \sin\vartheta + \frac{\partial}{\partial \vartheta}\left(\hat{p}_z \cos\vartheta\right)\right] x + f_1(\theta)$$

$$= \left(\hat{p}_z - \hat{p}_\vartheta\right) x \sin\vartheta + f_1(\vartheta)$$

$$\begin{cases} f_1(\theta) = C_1 \sin\vartheta \\ f_2(\vartheta) = C_2 \cos\vartheta \end{cases}$$

$$n_{x\vartheta} = \left[\left(\hat{p}_z - \hat{p}_\vartheta\right) x + C_1\right] \sin\vartheta$$

$$n_{xx} = \frac{1}{2a}\left(\frac{\partial p_\vartheta}{\partial \vartheta} + \frac{\partial^2 p_z}{\partial \vartheta^2}\right) x^2 - \frac{1}{a}\frac{\partial f_1(\theta)}{\partial \vartheta} x + f_2(\vartheta)$$

$$= \frac{1}{2a}\left[\frac{\partial}{\partial \theta}\left(\hat{p}_\vartheta \sin\vartheta\right) + \frac{\partial^2}{\partial \theta^2}\left(\hat{p}_z \cos\vartheta\right)\right] x^2 - \frac{1}{a}\frac{\partial}{\partial \theta}\left(C_1 \sin\vartheta\right) + C_2 \cos\vartheta$$

$$= \frac{1}{a}\left[\frac{1}{2}\left(\hat{p}_z - \hat{p}_\vartheta\right) x^2 + C_1 x + C_2\right] \cos\vartheta$$

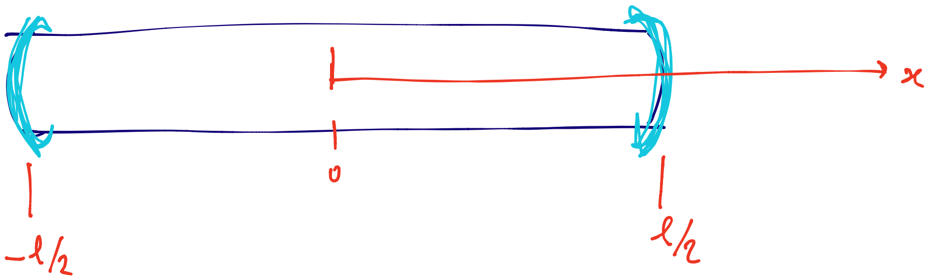
Gli spostamenti (in generale) sono [DIMOSTRARE]

$$Et\, u_x = \int n_{xx}\, dx - \nu \int n_{\theta\theta}\, dx$$

$$Et\, u_\theta = -\frac{1}{a} \iint \frac{\partial n_{xx}}{\partial \theta}\, dx\, dx + \frac{\nu}{a} \iint \frac{\partial n_{\theta\theta}}{\partial \theta}\, dx\, dx + 2(1+\nu) \int n_{x\theta}\, dx$$

$$Et\, u_z = \frac{1}{a} \iint \frac{\partial^2 n_{xx}}{\partial \theta^2}\, dx\, dx - \frac{\nu}{a} \iint \frac{\partial^2 n_{\theta\theta}}{\partial \theta^2}\, dx\, dx - 2(1+\nu) \int \frac{\partial n_{x\theta}}{\partial \theta}\, dx - \nu a\, n_{xx} + a\, n_{\theta\theta}$$

GUSCIO CILINDRICO APPOGGIATO CON DIAFRAMMI

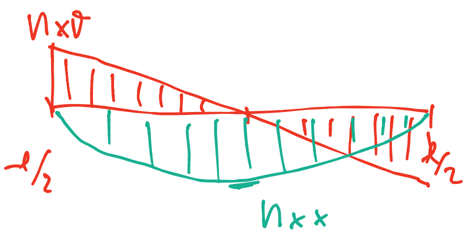



$$u_\theta\left(-\frac{\ell}{2}\right) = u_\theta\left(\frac{\ell}{2}\right) = 0$$

$$u_z\left(-\frac{\ell}{2}\right) = u_z\left(\frac{\ell}{2}\right) = 0$$

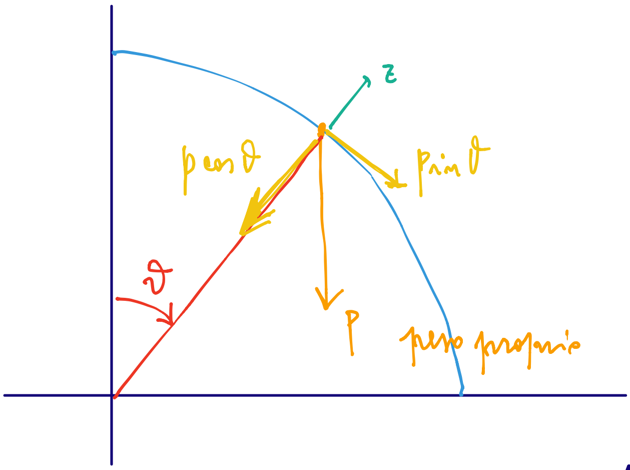
$$n_{xx}\left(-\frac{\ell}{2}\right) = n_{xx}\left(\frac{\ell}{2}\right) = 0$$

$$n_{x\theta}(0) = 0$$




$$p_\theta = p \sin\theta$$

$$p_z = -p\cos\theta$$




$$\hat{p}_\vartheta = p$$

$$\hat{p}_z = -p$$

$$\Downarrow$$

costanti di integrazione

$$C_1 = 0 \; ; \quad C_2 = -\frac{1}{8} l^2 \left(\hat{p}_z - \hat{p}_\vartheta\right) = \frac{1}{4} p l^2$$

$$n_{\vartheta\vartheta} = -a p \cos\vartheta$$

$$n_{x\vartheta} = -2 p x \sin\vartheta$$

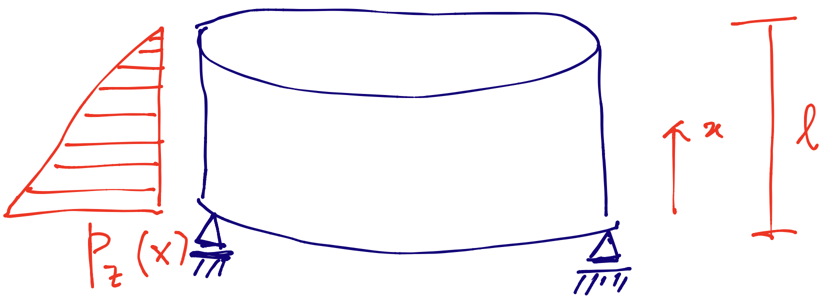
$$n_{xx} = -\frac{p l^2}{4a}\left[1 - \left(\frac{2x}{l}\right)^2\right]\cos\vartheta$$

ALCUNE RIFLESSIONI

① sforzo di taglio $n_{x\vartheta}$ → trave di lunghezza $l$ sottoposta a carico uniforme

② sforzo normale nella direzione $x$ $n_{xx}$ è distribuito lungo $x$ come il momento flettente della trave

# GUSCIO CIRCOLARE (CILINDRICO) SOGGETTO A CARICO ASSIALSIMMETRICO




① $p_z$ uniforme a $x$ fissato

② anche lo spostamento trasversale $u_z$ è uniforme a $x$ fissato

③ tutte le derivate rispetto a $\vartheta$ sono nulle

per $p_z(x,\vartheta) = p_z(x)$

le forze membranali sono

$$\begin{cases} n_{\vartheta\vartheta} = p_z(x)\, a \\ n_{x\vartheta} = 0 \\ n_{xx} = 0 \end{cases}$$

Le componenti di deformazione sono immediate

$$\begin{cases} \varepsilon_{xx} = \dfrac{1}{Et}\left(n_{xx} - \nu n_{\theta\theta}\right) = -\nu \dfrac{p_z(x)a}{Et} \\ \varepsilon_{\theta\theta} = \dfrac{1}{Et}\left(-\nu n_{xx} + n_{\theta\theta}\right) = \dfrac{p_z(x)a}{Et} \\ \gamma_{x\theta} = \dfrac{2(1+\nu)}{Et} n_{x\theta} = 0 \end{cases}$$

da cui deduciamo gli spostamenti

$$\begin{cases} u_x = \int \varepsilon_{xx}\, dx = -\nu \dfrac{a}{Et} \int p_z(x)\, dx + A \\ u_\theta = \int \left(\gamma_{x\theta} - \dfrac{1}{a}\dfrac{\partial u_x}{\partial \theta}\right) dx = B = 0 \quad \text{per simmetria} \\ u_z = a\left(\varepsilon_{\theta\theta} - \dfrac{1}{a}\dfrac{\partial u_\theta}{\partial \theta}\right) = \dfrac{p_z(x)\, a^2}{Et} \end{cases}$$

Applicazione a serbatoio d'acqua (densità $\gamma$)

$$p_z = \gamma(l - x) \Rightarrow n_{\theta\theta} = \gamma a (l - x)$$

$$\varepsilon_{xx}(x) = -\nu \frac{\gamma a}{Et}(l - x) \; ; \quad \varepsilon_{\theta\theta}(x) = \frac{\gamma a}{Et}(l - x)$$

$$u_x = -\frac{\nu a}{Et} \int p_z(x)\, dx + A$$

dalle condizioni al bordo $u_x(0) = 0 \Rightarrow A = 0$

$$u_x = -\frac{\nu a}{Et} \int \gamma (l - x)\, dx$$

$$= -\frac{\nu a}{Et} \gamma \left( l x - \frac{x^2}{2} \right)$$

$$u_z = \frac{\gamma a^2}{Et} (l - x$$