Introduction into Cosserat-type theories of beams, plates and shells with applications. Classic theories.

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Materials & Structures under Consideration

Classification

Starting Point

Material science classifies structural materials into three categories

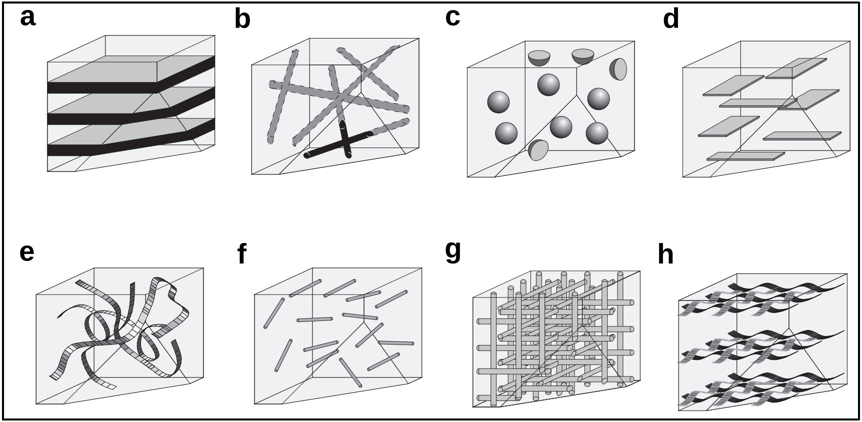
- metals,
- ceramics, and
- polymers

It is difficult to give an exact assessment of the advantages and disadvantages of these three basic material classes, because each category covers whole groups of materials within which the range of properties is often as broad as the differences between the three material classes.

Characteristic Properties

- Mostly metallic materials are of medium to high density. They have good thermal stability and can be made corrosion-resistant by alloying. Metals have useful mechanical characteristics and it is moderately easy to shape and join. For this reason metals became the preferred structural engineering material, they posed less problems to the designer than either ceramic or polymer materials.
- Ceramic materials have great thermal stability and are resistant to corrosion, abrasion and other forms of attack. They are very rigid but mostly brittle and can only be shaped with difficulty.
- Polymer materials (plastics) are of low density, have good chemical resistance but lack thermal stability. They have poor mechanical properties, but are easily fabricated and joined. Their resistance to environmental degradation, e.g. the photomechanical effects of sunlight, is moderate.

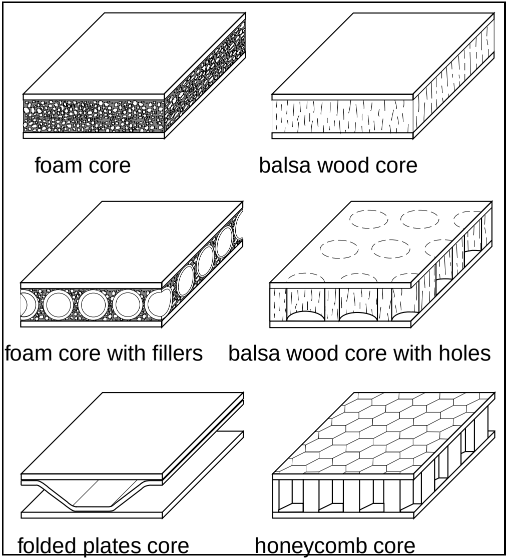
Classification of Composites[1]



a Laminate, **b** irregular reinforcement, **c** reinforcement with particles, **d** reinforcement with plate strapped particles, **e** random arrangement of continuous fibres, **f** irregular reinforcement with short fibres, **g** spatial reinforcement, **h** reinforcement with surface tissues

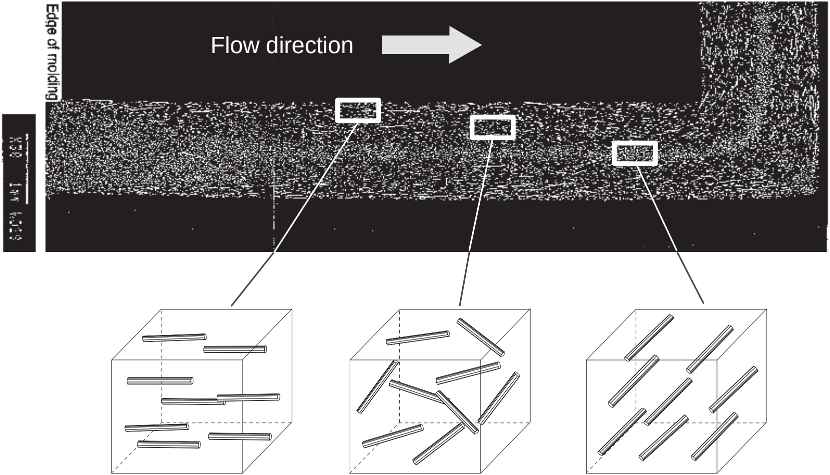
[1]Altenbach et al. Mechanics of Composite Structural Elements, Springer 2004

Sandwich Materials with Solid and Hollow Cores[2]



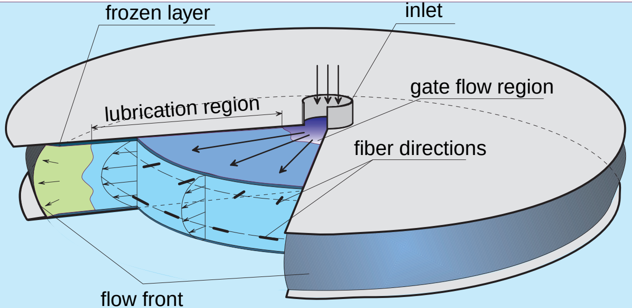
[2] Altenbach et al. Mechanics of Composite Structural Elements, Springer 2004

Short Fibre Reinforced Composite[3]

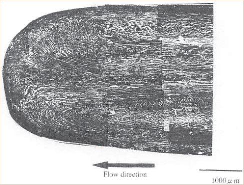


[3] Saito et al. Material Science and Engineering, A285:280-287, 2000

Injection Molding

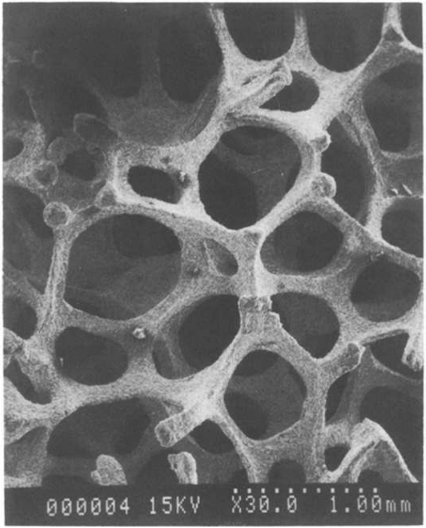


Sketch of a radial flow between parallel plates

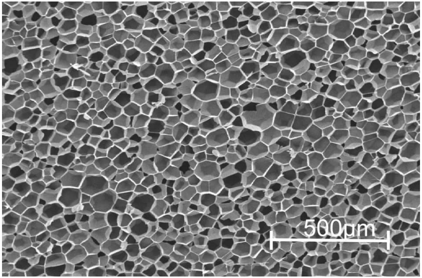


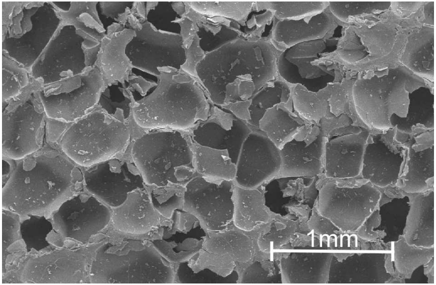
Cross section of a shot specimen, fabricated by injection molding, *Semba and Hamada, 1999*

Copper Foams



Closed-cell Foams[4]

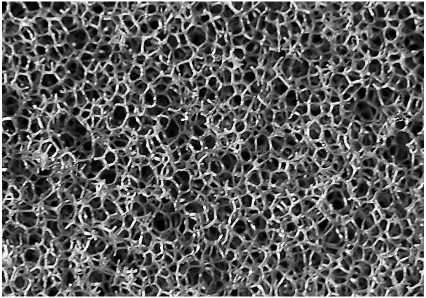


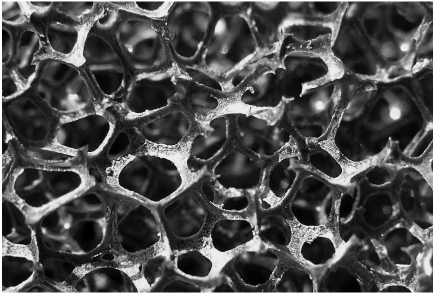


Closed-cell polymeric foams with various densities

[4]Kraatz (2007), DKI, Darmstadt, Germany

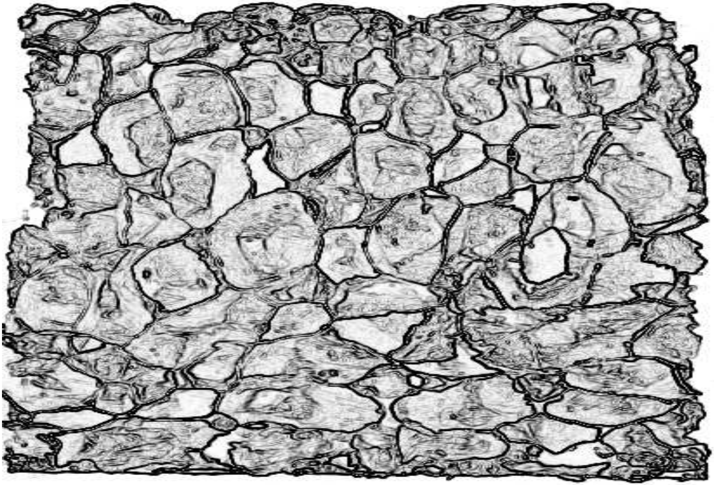
Open-cell Foams





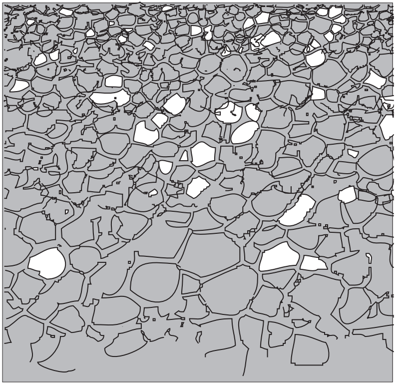
Open-cell foams with various densities

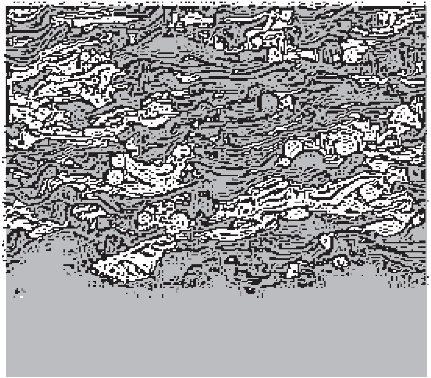
Functionally Graded Material (FGM)



Inhomogeneous Microstructure of a FGM

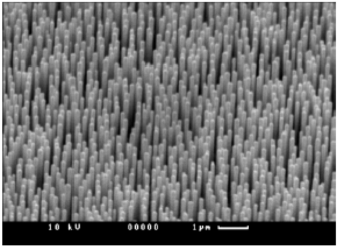
Examples of FGMs

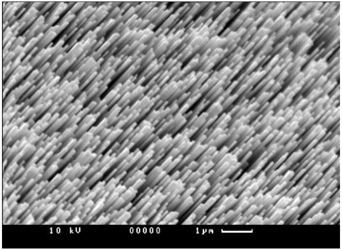


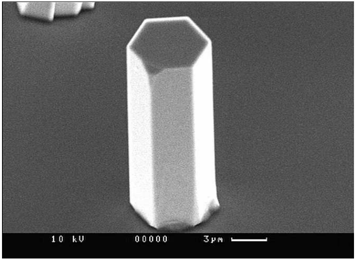


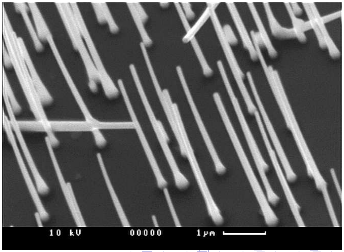
Inhomogeneous microstructure:
Foam (left), Thermal coating (right)

Nanostructures

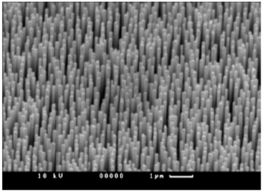


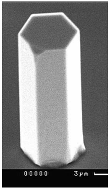


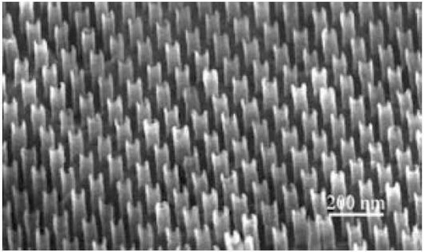


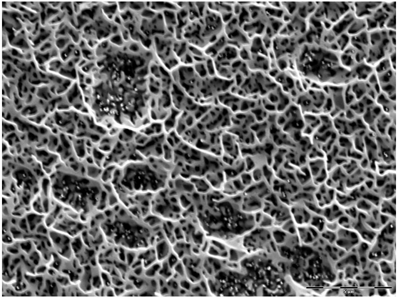


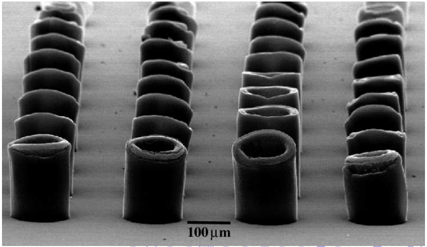
Nanostructures







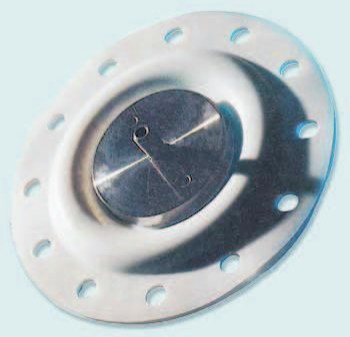




Applications

Sandwich Application in Automotive Industries

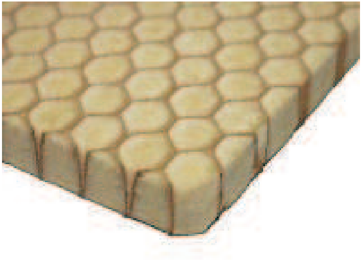
Sandwich Application in Chemical Industries[5]



Sandwich-Membrane (PTFE)

[5] http://www.elringklinger-kunststoff.de/pages/article/article0601chem.html

Structural Honeycomb - Foam Filled[6]



[6]http://www.nida-core.com/english/nidaprod_honey_foam.htm

Door made of Steel-Honeycomb[7]

Honeycomb has the highest strength to weight ratio (in a sandwich form), of any known material

[7] http://www.designandsupply.co.uk/steeldoors_types_cores.html

Short Fibre Composites Applications

Automotive

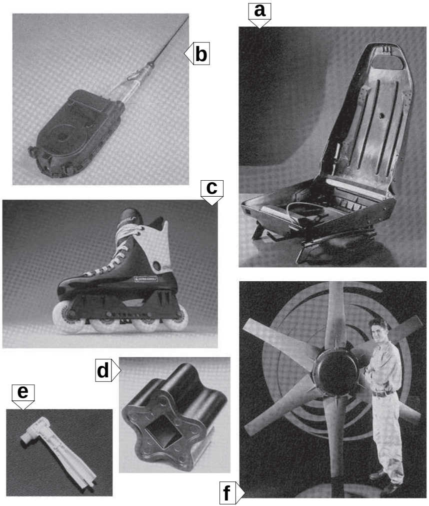
Civil engineering

Sort fibre reinforced composites

Sports

Sanitary applications

Short Fibre Composites Applications[8]



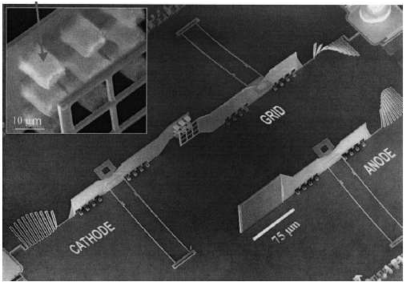
a sport car seat back
PA66, 50% long glass fibres
b housing car radio antenna
PA66, 25% glass fibres
c In-line roller skate frame
PA66, 40% long glass fibres
d gear pump impeller
epoxy 40% glass fibres
e dental equipment bevel gears
PTFE-PA66 30% glass faibres
f Cooling fan hub
and blade
PA66, 50% long glass fibres

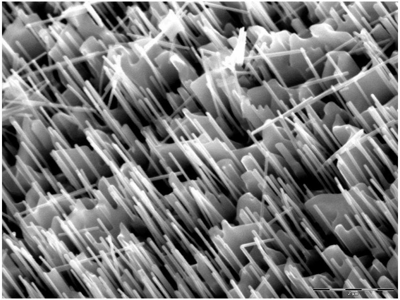
[8] Jones, R.M. Guide to Short Fiber Reinforced Plastics. München: Hanser, 1998

Composite Applications - Telecabine[9]

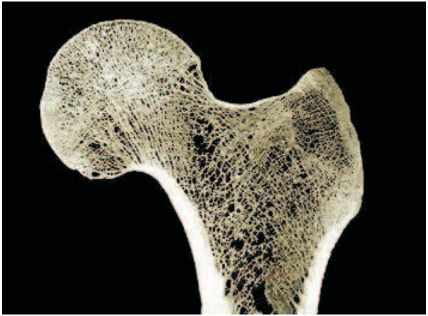
[9] http://www.poma.net/deutsch/produits/sporthiver/telecabine/4places/main.htm

Nanostructure Applications



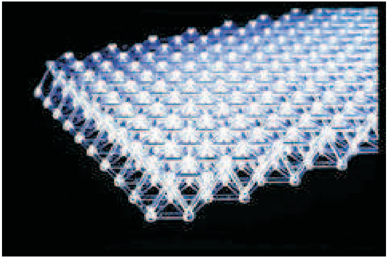


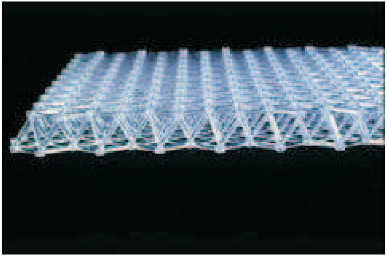
Femur[10]



[10] http://www.seilnacht.tuttlingen.com/Minerale/Seeigel.htm

Lattice Structure



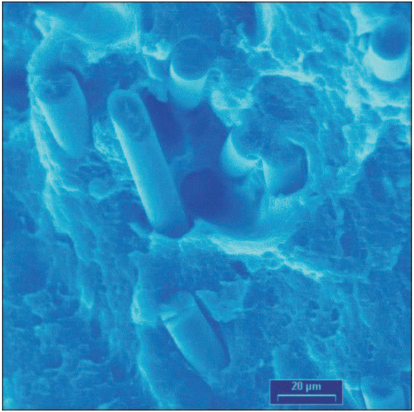


FGM Application[11]

FGM European Virtual Institute on Knowledge-based Multifunctional Materials AISBL

[11] http://www.kmm-vin.eu/Research/FunctionallyGradedMaterials/tabid/68/Default.aspx

Composite[12]



Fibre-reinforced Composite

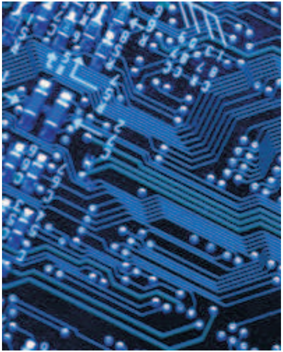
[12] http://www.uni-magdeburg.de/iwf/labore/wt/13/imgal.html

Composite[13]

US Air Force F-117 Nighthawk

[13] http://de.wikipedia.org/wiki/Flugzeug

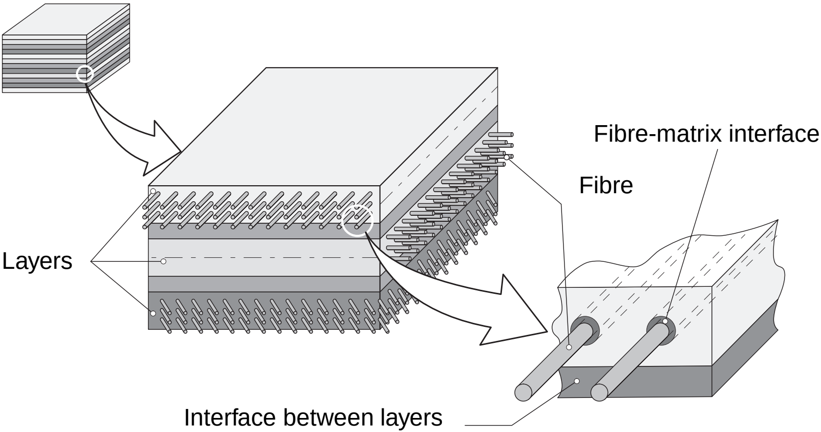
Computerchip[14]



[14] http://pressetext.de/news/061023004/computerchip-entwickler-kaempfen-mit-kuehlungsproblemen/

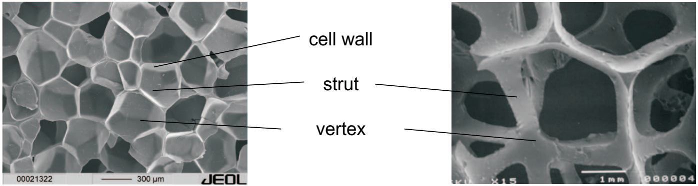
Hierarchical Modeling

Laminated Plates



Open- and closed-cell Foams

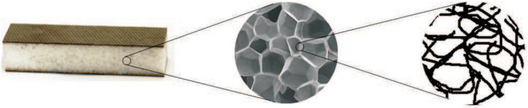
closed cell foam | **open cell foam**



Rohacell 51WF; IWMH | open cell PU-foam; Mills2000

- microscopic investigations,
- Röntgen-Tomography,
- Geometrical properties as statistic distributions

Three Scales



Fundamentals of the Plate Theory

Classical Plate Theories

Basic Problem in Civil & Mechanical Engineering

Analysis of the strength, the vibration behavior and the stability of structures with the help of structural models.

The structural models can be classified by their

- spatial dimensions
- loadings
- kinematical and/or statical hypotheses

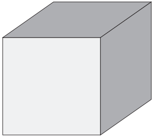
The structural model of a whole structure includes the interaction of all parts.

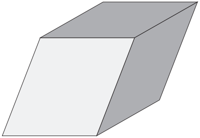
Three-dimensional Structural Elements

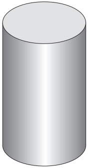
- **The three spatial dimensions have the same order, no predominant direction for the dimensions exists.**

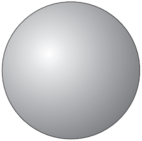
Typical examples of geometrical simple, compact structural elements in theory of elasticity are:

- Cube
- Prisma
- Cylinder
- Sphere







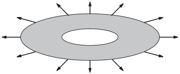


Two-dimensional Structural Elements

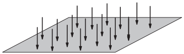
- **Two spatial dimensions have the same order, the third, which is related to the thickness is much smaller.**

Typical examples of surface structural elements in structural mechanics are:

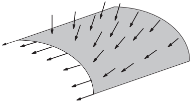
- Discs



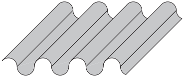
- Plates



- Shells



- Folded Structures

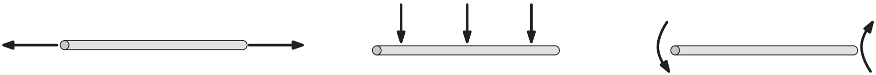


One-dimensional Structural Elements

- **Two spatial dimensions, which can be related to the cross-section, have the same order. The third dimension, which is related to the length of the structural element, has a much larger order in comparison with the cross-section dimensions.**

Typical examples in engineering mechanics are:

- Rods
- Beams
- Torsion beam



Thin-walled Structural Elements

Thin-walled light-weight profile structures require an extension of the classical structural models:

- **If the spatial dimensions are of significantly different order and the thickness of the profile is small in comparison to the other cross-section dimensions, and the cross-section dimensions are much smaller in comparison to the length of the structure one can introduce quasi-one-dimensional structural elements. The suitable theories are**
  - **the thin-walled beam approach (Vlasov-Theory) and**
  - **the semi-membrane theory or generalized beam theory**

2D Structures - Definition

Definition:

A two-dimensional load-barring structural element is a model for the analysis in Engineering/Structural Mechanics with two geometrical dimensions, which are of the same order and which are significantly larger as the third (thickness) direction.

Mathematical Consequence:

Instead of a three-dimensional problem, which is presented by a system of partial differential equations, one can analyze a two-dimensional problem. The transition from the three-dimensional to the two-dimensional problem is not simple, but the solution effort decreases significantly.

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Applications

- **Examples of model classes:**
  - thin homogeneous plates
  - thin inhomogeneous plates (laminates, sandwiches)
  - plates with structural anisotropy
  - moderately thick homogeneous plates
  - folded plates
  - membranes
  - biological membranes
  - nanotubes
- **Applications:**
  - space and aircraft industries
  - automotive industries
  - shipbuilding industries
  - vehicle systems
  - civil engineering
  - medicine

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Literature

- **Some review articles and monographs:**
  - P.M. Naghdi, 1972
  - E.I. Grigolyuk, 1972, 1988
  - I.F. Obrazcov, 1983
  - E. Reissner, 1985
  - G. Wempner, 1989
  - A.K. Noor, 1989, 1990
  - J.N. Reddy, 1990
  - H. Irschik, 1992
- **Actual Conferences:**
  - EUROMECH *Critical Review of the Theories of Plates and Shells, New Applications*, Bremen 2002
  - *Shell Structures Theory & Applications*, Jurata 2009
  - IUTAM *Relation of Shell, Plate, Beam and 3D Models*, Tbilisi 2007

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Formulation Principles

Plate equations can be deduced:

- Starting from 3D continuum
- Starting from 2D continuum

If one starts from the 3D continuum - two possibilities:

- the use of hypotheses
- the use of mathematical approaches

All methods have advantages and disadvantages!

Hypotheses based theories are preferred by the engineers!

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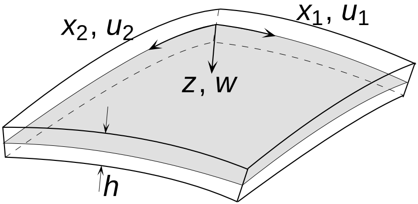
Displacement Approximations

$$u_i = (u_\alpha, w)$$

$$\alpha, \beta = 1, 2$$

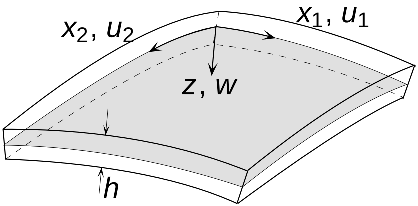


- **Kirchhoff (1850)**
  $$u_\alpha(x_\beta, z) = u^0_\alpha(x_\beta) - z w_{,\alpha}(x_\beta), \quad w(x_\beta, z) = w(x_\beta)$$
- **Hencky, Bollé (1947), Mindlin (1951)**
  $$u_\alpha(x_\beta, z) = u^0_\alpha(x_\beta) + z \varphi_\alpha(x_\beta), \quad w(x_\beta, z) = w(x_\beta)$$
- **Levinson (1981), Reddy (1984)**
  $$u_\alpha(x_\beta, z) = u^0_\alpha(x_\beta) - [w_{,\alpha}(x_\beta) + \varphi_\alpha(x_\beta)]\frac{4z^3}{3h^2}, \quad w(x_\beta, z) = w(x_\beta)$$
- **Altenbach, Meenen (1999)**
  $$u_\alpha(x_\beta, z) = u^q_\alpha(x_\beta)\phi^q(z) + w^q_{,\alpha}(x_\beta)\psi^q(z), \quad w(x_\beta, z) = w(x_\beta)^q \chi^q(z)$$

Displacement Approximations

$u_i = (u_\alpha, w)$

$\alpha, \beta = 1, 2$



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- **Hencky, Bollé (1947), Mindlin (1951)**
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- **Levinson (1981), Reddy (1984)**
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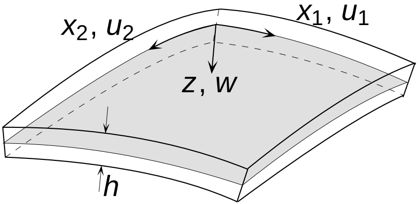

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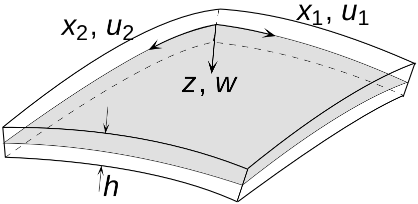

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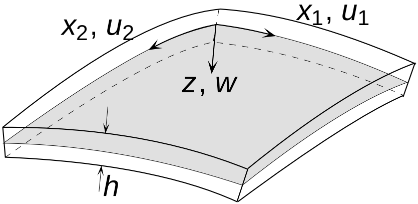

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  - Lo, Christensen & Wu (1977)
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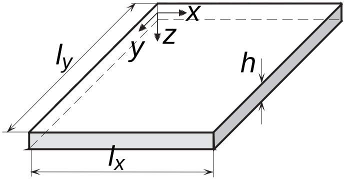
Kirchhoff's Model



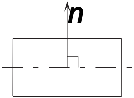
Kirchhoff plate

Kinematical hypotheses

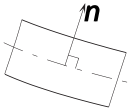
$h/\min(l_x, l_y) < 0.1$,

$w/h < 0.2$

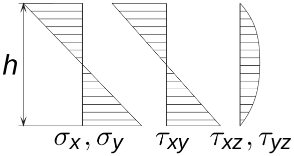
$\varepsilon_z \approx 0$



$\gamma_{xz}, \gamma_{yz} \approx 0$



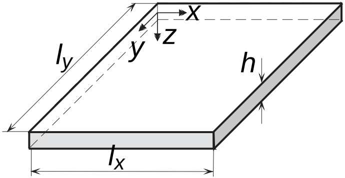
Stress distribution



Normal stresses σ_x, σ_y and shear stress τ_{xy} linear over h

τ_{xz}, τ_{yz} parabolic

Mindlin's Model



Mindlin plate

Kinematical hypotheses

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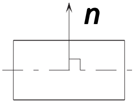
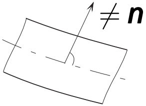
$w/h < 0.2$

$\varepsilon_z \approx 0$



$\gamma_{xz}, \gamma_{yz} \approx$ const



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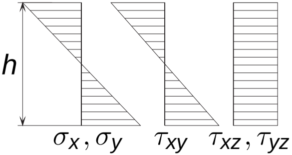
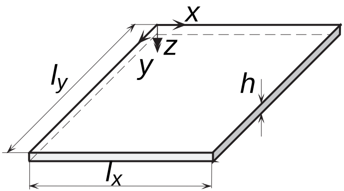


Normal stress σ_x, σ_y and shear stress τ_{xy} linear over h

τ_{xz}, τ_{yz} constant over h

Membrane Model

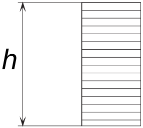


Membrane

Assumptions
$h \ll \min(l_x, l_y)$,
$w/h \geq 0.5$

$\tau_{xy}, \tau_{xz}, \tau_{yz}, \sigma_z \approx 0$

Stress distribution



σ_x, σ_y
(tension stresses)

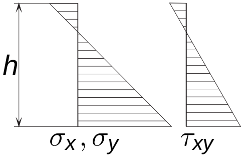
No shear stresses!

von Kármán's Model

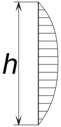
von Kármán plate

Stress distribution

Assumptions
$h/\min(l_x, l_y) < 0.1$,
$0.2 < w/h < 5$

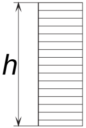


shear rigid
$\varepsilon_z, \gamma_{xz}, \gamma_{yz} \approx 0$



shear rigid
model
τ_{xz}, τ_{yz}

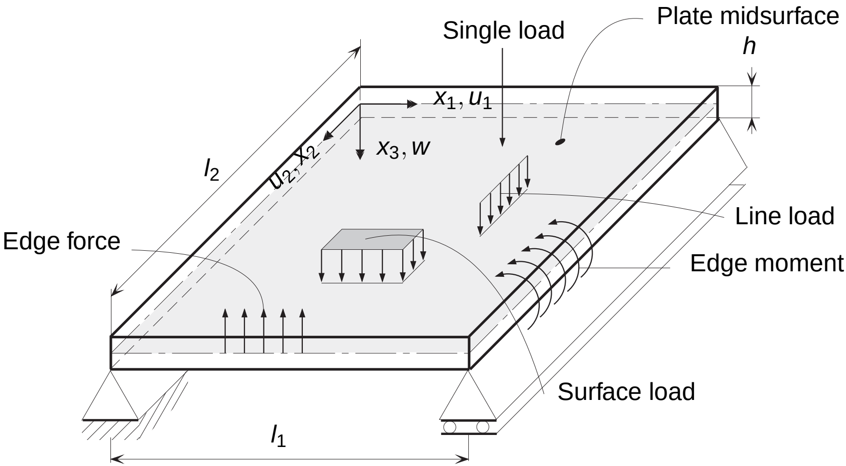
shear deformable
$\varepsilon_z \approx 0, \gamma_{xz}, \gamma_{yz} \approx \text{const}$



shear deformable
model
τ_{xz}, τ_{yz}

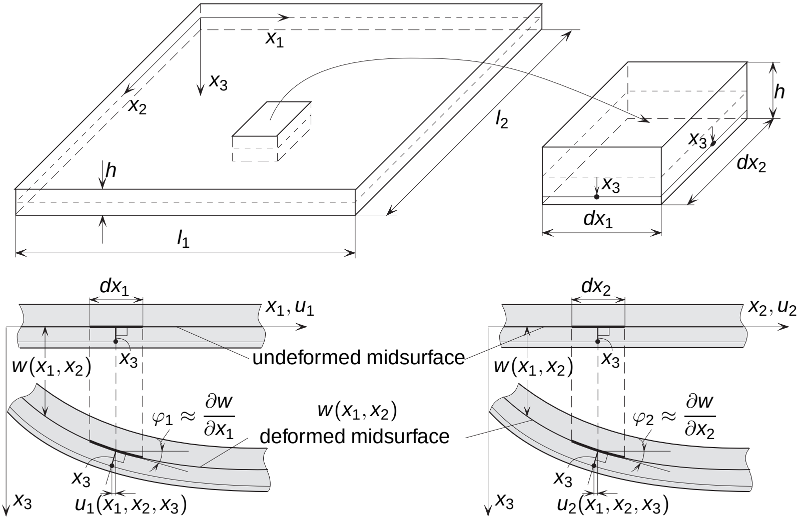
Kirchhoff Plate (I)

Loading



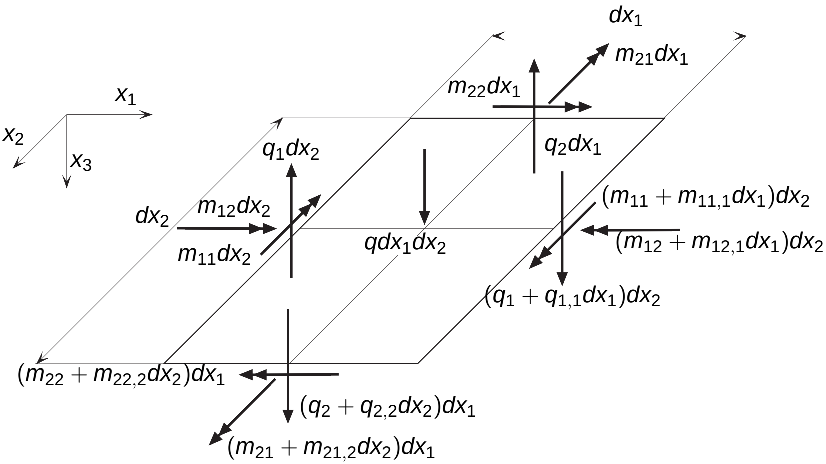
Kirchhoff Plate (II)

Kinematical relations

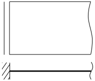
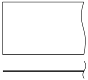
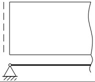
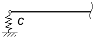
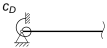
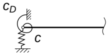


Kirchhoff Plate (III)

Stress resultants

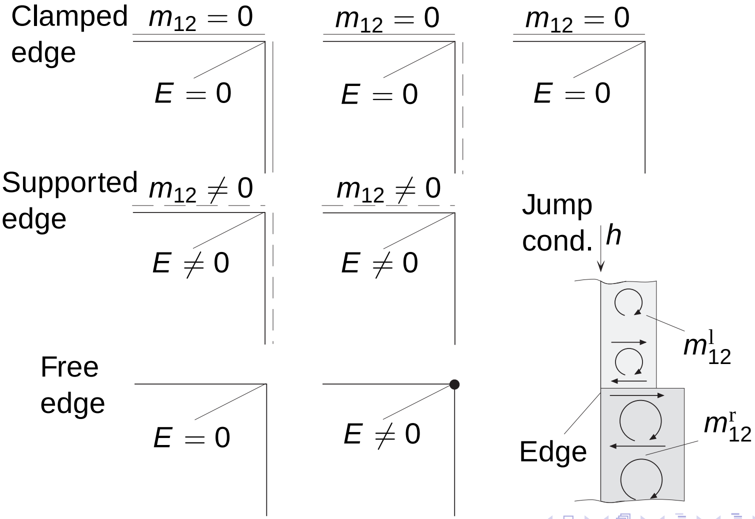


Kirchhoff Plate (IV) - Boundary Conditions

Type	Symbol	Description
Rigid clamped		kinematical b.c.: deflection and rotation are equal to zero
Free boundary		static b.c.: all resultants are equal to zero
Fixes support		mixed b.c.: deflection and bending moment are equal to zero
Elastic support		mixed b.c.: bending moment is zero and the force is proportional to the deflections
Elastic clamped		mixed b.c.: deflection is zero and moment is proportional to the cross section rotation
Elastic support - clamped		mixed b.c.: force is proportional to the deflection and the moment is proportional to the rotation

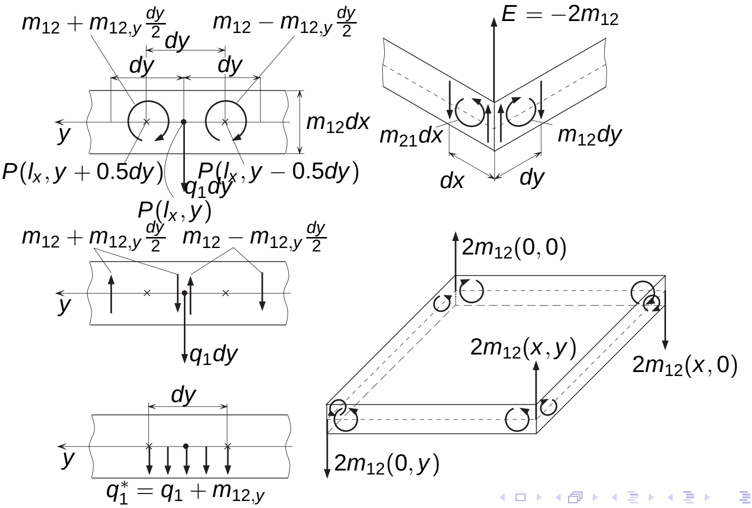
Kirchhoff Plate (V)

Kirchhoff's Edge Forces



Kirchhoff Plate (VI)

Kirchhoff's Edge Forces



Kirchhoff Plate (VII)

Rectangular plate with constant bending stiffness

- Bending equation for simply supported plate

$$K\Delta\Delta w(x_1,x_2) = q(x_1,x_2)$$

- Bending equation for elastically supported plate

$$K\Delta\Delta w(x_1,x_2) = q(x_1,x_2) - cw(x_1,x_2)$$

- Bending vibration equation for simply supported plate

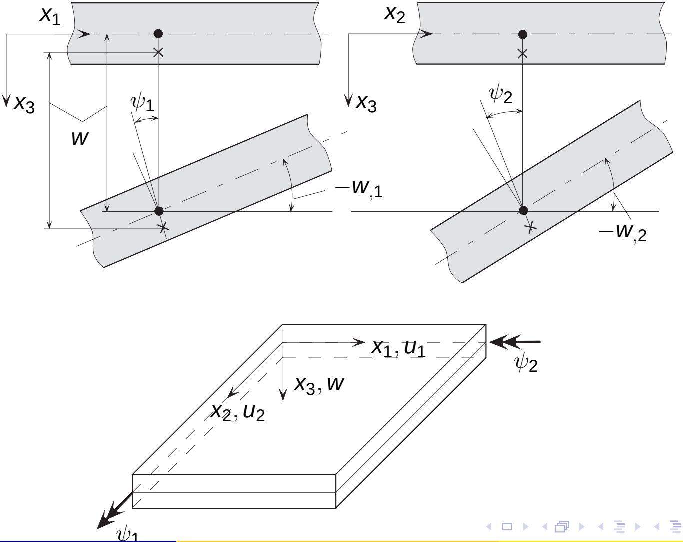
$$K\Delta\Delta w(x_1,x_2,t) + \rho h \ddot{w}(x_1,x_2,t) = q(x_1,x_2,t)$$

- Bending vibration equation for elastically supported plate

$$K\Delta\Delta w(x_1,x_2,t) + \rho h \ddot{w}(x_1,x_2,t) = q(x_1,x_2,t) - cw(x_1,x_2,t)$$

Mindlin Plate (I)

Kinematics



Mindlin Plate (II)

Basic equations of the Mindlin theory

$$
\begin{aligned}
Gh_S(\Delta w + \Phi) + q &= \rho h \ddot{w}, \\
\frac{K}{2}[(1-\nu)\Delta\psi_1 + (1+\nu)\Phi_{,1}] - Gh_S(\psi_1 + w_{,1}) &= \frac{\rho h^3}{12}\ddot{\psi_1}, \\
\frac{K}{2}[(1-\nu)\Delta\psi_2 + (1+\nu)\Phi_{,2}] - Gh_S(\psi_2 + w_{,2}) &= \frac{\rho h^3}{12}\ddot{\psi_2}
\end{aligned}
$$

With $(\psi_{1,1} + \psi_{2,2} = \Phi)$ on gets

$$
\left(K\Delta - Gh_s - \frac{\rho h^3}{12}\frac{\partial^2}{\partial t^2}\right)\Phi = Gh_s \Delta w
$$

and after elimination of Φ

$$
\left(\Delta - \frac{\rho h}{Gh_S}\frac{\partial^2}{\partial t^2}\right)\left(K\Delta - \frac{\rho h^3}{12}\frac{\partial^2}{\partial t^2}\right)w + \rho h\frac{\partial^2 w}{\partial t^2} = \left(1 - \frac{K}{Gh_S}\Delta + \frac{\rho h^3}{12Gh_S}\frac{\partial^2}{\partial t^2}\right)q
$$

Mindlin Plate (III)

Special cases of the Mindlin theory

$$\left(\Delta - \frac{\rho h}{G h_S}\frac{\partial^2}{\partial t^2}\right)\left(K\Delta - \frac{\rho h^3}{12}\frac{\partial^2}{\partial t^2}\right)w + \rho h\frac{\partial^2 w}{\partial t^2} = \left(1 - \frac{K}{G h_S}\Delta + \frac{\rho h^3}{12 G h_S}\frac{\partial^2}{\partial t^2}\right)q$$

- Neglecting the rotational inertia ($\rho h^3 \longrightarrow 0$)

$$K\left(\Delta - \frac{\rho h}{G h_S}\frac{\partial^2}{\partial t^2}\right)\Delta w + \rho h\frac{\partial^2 w}{\partial t^2} = \left(1 - \frac{K}{G h_S}\Delta\right)q$$

- Neglecting the shear stiffness ($G h_S \longrightarrow \infty$)

$$\left(K\Delta - \frac{\rho h^3}{12}\frac{\partial^2}{\partial t^2}\right)\Delta w + \rho h\frac{\partial^2 w}{\partial t^2} = q$$

- Neglecting both the rotational inertia and the shear stiffness

$$K\Delta\Delta w + \rho h\frac{\partial^2 w}{\partial t^2} = q$$

Comparison of both Models

Shear deformable plate	Shear rigid plate
$K\Delta\Delta w^* = q,\ \frac{1-\nu}{2}\frac{K}{Gh_s}\Delta\Psi - \Psi = 0$ with $\Psi = (\psi_{2,1} - \psi_{1,2})$	$K\Delta\Delta w = q,\ \Psi = 0$
$w = w^* - \frac{K}{Gh_s}\Delta w^*$	$w^* = w$
$\psi_1 = -w^*_{,1} - \frac{1-\nu}{2}\frac{K}{Gh_s}\Psi_{,2}$	$\psi_1 = -w_{,1}$
$\psi_2 = -w^*_{,2} + \frac{1-\nu}{2}\frac{K}{Gh_s}\Psi_{,1}$	$\psi_2 = -w_{,2}$
$m_{11} = -K\left[w^*_{,11} + \nu w^*_{,22} + \frac{(1-\nu)^2}{2}\frac{K}{Gh_s}\Psi_{,12}\right]$	$m_{11} = -K(w_{,11} + \nu w_{,22})$
$m_{22} = -K\left[w^*_{,22} + \nu w^*_{,11} + \frac{(1-\nu)^2}{2}\frac{K}{Gh_s}\Psi_{,12}\right]$	$m_{22} = -K(w_{,22} + \nu w_{,11})$
$m_{12} = -K\left[(1-\nu)w^*_{,12} - \frac{(1-\nu)^2}{4}\frac{K}{Gh_s}(\Psi_{,11} + \Psi_{,22})\right]$	$m_{12} = -K(1-\nu)w_{,12}$
$q_1 = -K\left[(\Delta w^*)_{,1} + \frac{1-\nu}{2}\Psi_{,2}\right]$	$q_1 = -K(\Delta w)_{,1}$
$q_2 = -K\left[(\Delta w^*)_{,2} + \frac{1-\nu}{2}\Psi_{,1}\right]$	$q_2 = -K(\Delta w)_{,2}$

Non-classical Approaches

Direct Approach

A priori introduction of an two-dimensional deformable surface

- W. Günther (1961)
- Green et al. (1964)
- Naghdi (1972)
- Rothert (1973)
- Zhilin (1976/82,2007)
- Pal'mov (1982)
- Rubin (2000)
- Altenbach & Eremeyev (2008/09)

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Reference

"‘. . . everyone, who is thinking about the foundations of Continuum Mechanics, will attend the world of images of the COSSERAT brothers."’

H. SCHAEFER

from:

Das Cosserat-Kontinuum, *ZAMM* **47**(1967)8, 485 - 498

(published as a summarizing report)

Plenary Lecture at the GAMM Conference in March 1967 in Zurich

History - 3D

- Leonard Euler - introduction of the moment vector as independent quantity
- Eugène et François Cosserat
  - Sur la théorie de l'élasticité, Ann. Toulouse **10**, 1-116 (1896)
  - Théorie des corps déformables, Paris, Herman, 1909
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