On the theory of plates and shells at the nano- and microscales considering surface effects

Victor A. Eremeyev

Rzeszow University of Technology, Rzeszów, Poland

Università degli Studi di Cagliari, Cagliari, June, 2017

Contents

Contents

Contents

Contents

Contents

Phenomena

- The development of nanotechnologies extends the field of application of the classical or non-classical theories of plates and shells towards the new thin-walled structures.
- In general, modern nanomaterials have physical properties which are different from the bulk material.
- The classical linear elasticity can be extended to the nanoscale by implementation of the theory of elasticity taking into account the surface stresses, cf. Duan et al. (2008) among others.
- In particular, the surface stresses are responsible for the size-effect, that means the material properties of a specimen depend on its size.
- For example, Young's modulus of a cylindrical specimen increases significantly, when the cylinder diameter becomes very small.
- The surface stresses are the generalization of the scalar surface tension which is well-known phenomenon in the theory of capillarity.

Our Aim is

- to discuss the effective constitutive equations for surface stresses taking into account complex structure of surface and/or surface coatings;
- to analyze of the influence of surface effects on the effective properties of materials such as the effective bending stiffness of plates or the stiffness of rods.

Surface elasticity models

- Based on additional surface (2D) constitutive equations. After Laplace (1805) and Young (1806).
- Based on unified gradient-type models. After van der Waals (1893) and Korteweg (1901).
- With sharp interface or
- with interfacial layer.

Surface Elasticity

- The investigations of the surface phenomena were initiated by Laplace (1805), Young (1806) & Gibbs (1875-1878).
- Works taking into account the surface stresses
  - Gurtin & Murdoch (1975)
  - Podstrigach & Povstenko (1985)
  - Steigmann & Ogden (1999)
- Residual surface stresses
  - Gurtin, Markenscoff, and Thurston (1976);
  - Wang and Feng (2007);
  - Wang and Zhao (2009).
- FEM realization
  - Javili and Steinmann (2009–2013).
- Reviews
  - Orowan (1970)
  - Podstrigach & Povstenko (1985)
  - Finn (1986)
  - Rusanov (2005)
  - Duan, Wang & Karihaloo (2008)
  - Wang et al. (2011)

Surface tension

Surface tension may be useful

Influence of Surface Stresses

Phase transitions

Nucleation, crystal growth, etc.

Fracture mechanics

Griffith criterion, Effective surface energy density, Line tension as a energy of a dislocation core

Mechanics of porous media

Nanoporous materials can be made stiffer than non-porous counterparts by surface modification

Other problems

Surface diffusion, Surface waves.

Influence of Surface Stresses

Phase transitions

Nucleation, crystal growth, etc.

Fracture mechanics

Griffith criterion, Effective surface energy density, Line tension as a energy of a dislocation core

Mechanics of porous media

Nanoporous materials can be made stiffer than non-porous counterparts by surface modification

Other problems

Surface diffusion, Surface waves.

Influence of Surface Stresses

Phase transitions

Nucleation, crystal growth, etc.

Fracture mechanics

Griffith criterion, Effective surface energy density, Line tension as a energy of a dislocation core

Mechanics of porous media

Nanoporous materials can be made stiffer than non-porous counterparts by surface modification

Other problems

Surface diffusion, Surface waves.

Influence of Surface Stresses

Phase transitions

Nucleation, crystal growth, etc.

Fracture mechanics

Griffith criterion, Effective surface energy density, Line tension as a energy of a dislocation core

Mechanics of porous media

Nanoporous materials can be made stiffer than non-porous counterparts by surface modification

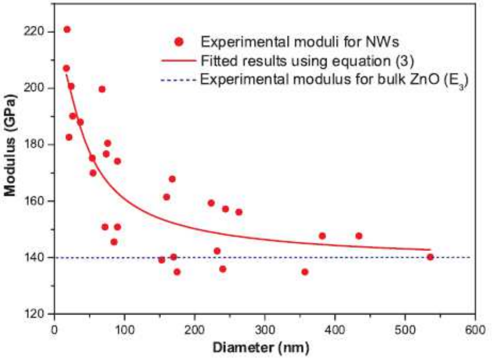
Other problems

Surface diffusion, Surface waves.

Experimental Observations (I)

Surface stresses → size effect

Young's modulus, experimental data: eigenfrequencies of nanowires[a]

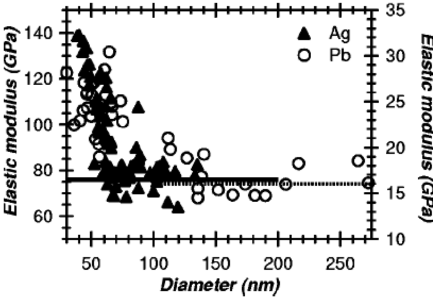


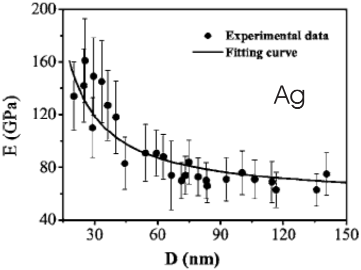
[a]Chen et al. (2006)

Experimental Observations (II)

Size effect

Young's modulus: bending of nanobeams made of Ag, Pb[a,b]



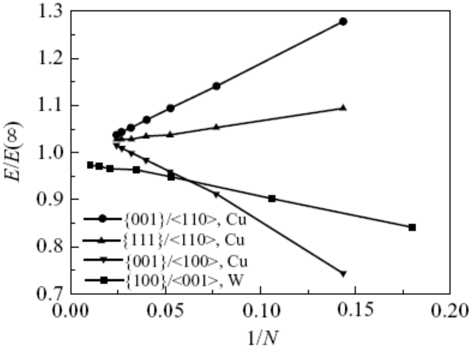


[a] Cuenot et al. (2004)
[b] Jing et al. (2006)

Experimental Observations (III)

Size effect

Young modulus: bending of nanoplates[a]
(molecular dynamics estimations)

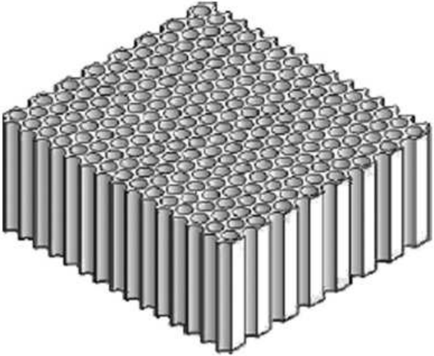


[a]Wang et al. (2006)

Experimental Observations (IV)

Size effect

Nanoporous materials can be made stiffer[a]



[a] Duan et al. (2005, 2008)

Experimental Observations (V)

Size effect

Dependence of the effective moduli on the size of pores[a]

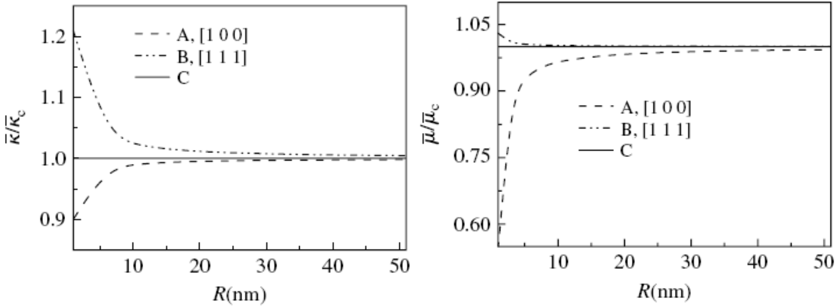


FIG. 5.1 Effective bulk modulus as a function of void radius ($f = 0.3$). A: $\kappa_s = -5.457$ N/m, $\mu_s = -6.2178$ N/m for the surface [1 0 0]; B: $\kappa_s = 12.932$ N/m, $\mu_s = -0.3755$ N/m for the surface [1 1 1]; C: the classical results without the surface stress effect. Reprinted from Duan et al. (2005b)

[a] Duan et al. (2005)

Scaling law for nano-structured materials

Wang et al. (2006), Duan et al. (2008): Dependence on the size length L.

Let F be some property, i.e. Young's modulus, temperature of melting, etc. Then we assume

$$\frac{F(L)}{F(\infty)} = 1 + \alpha \frac{l_{in}}{L} + O\left(\frac{l_{in}}{L}\right)^2$$

Example: temperature of melting of a nanoparticle of radius R are

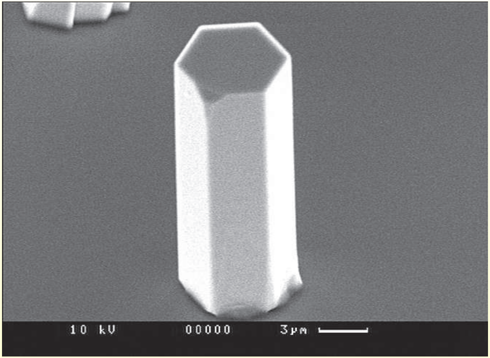
$$\frac{T(R)}{T(\infty)} = 1 - 2\frac{l_{in}}{R}$$

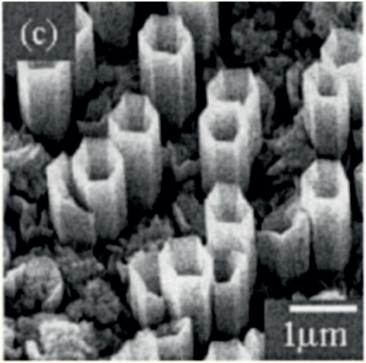
Here l_{in} is a characteristic length, usually $l_{in} = 2...20$ nm.

What type of surface may we have?

- Perfect and
- Non-Perfect. Is it really surface?

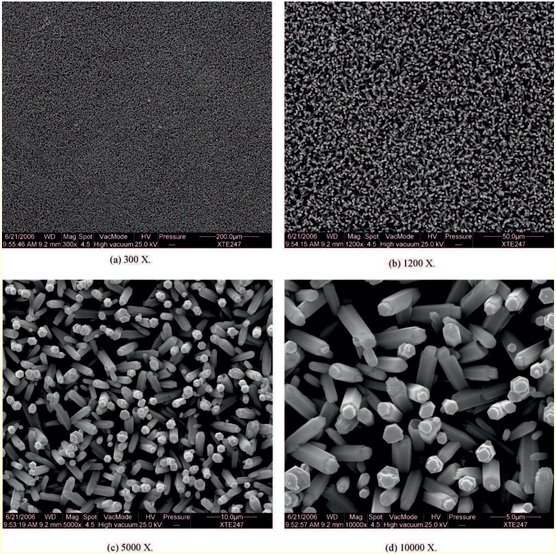
Perfect surfaces – planes and mathematical surfaces





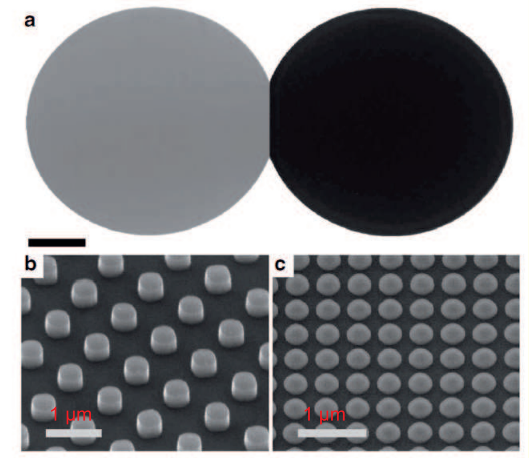
ZnO crystal, ZnO nanotubes, etc.

Non-Perfect surface



ZnO nanoarray, Xingfa Ma et al. *Materials Research Bulletin*, 2008.

Broadband omnidirectional antireflection coating

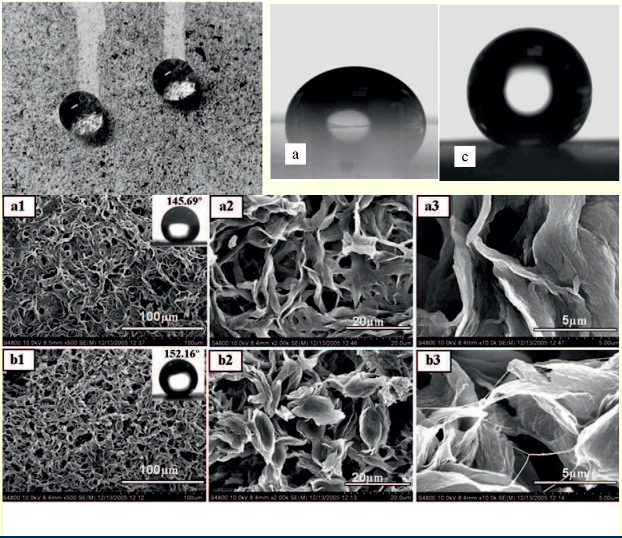


”Black Silicon” for solar cells.

Spinelli et al. *Nature Communications*, 2011.

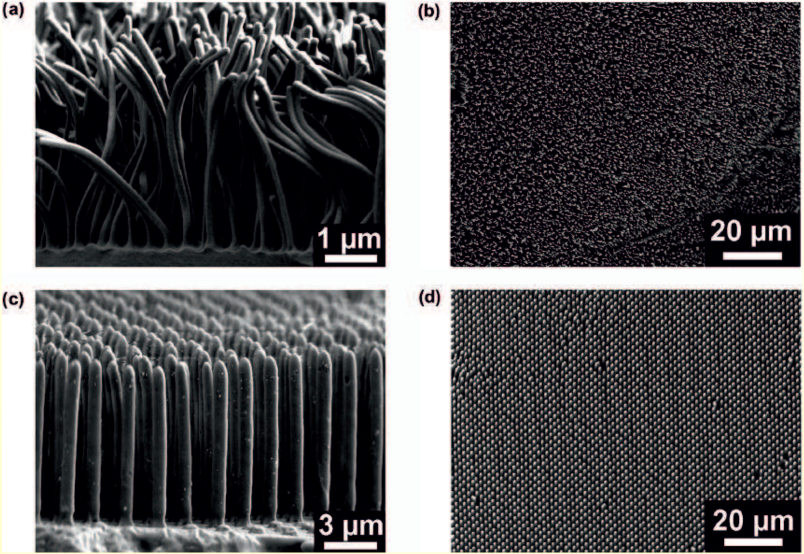
Self-cleaning polymer coating

X. Kang et al. *Appl. Surface Sci*, 2007;
P. F. Rios et al. *J. Adhesion Sci. Technol.*, 2007.



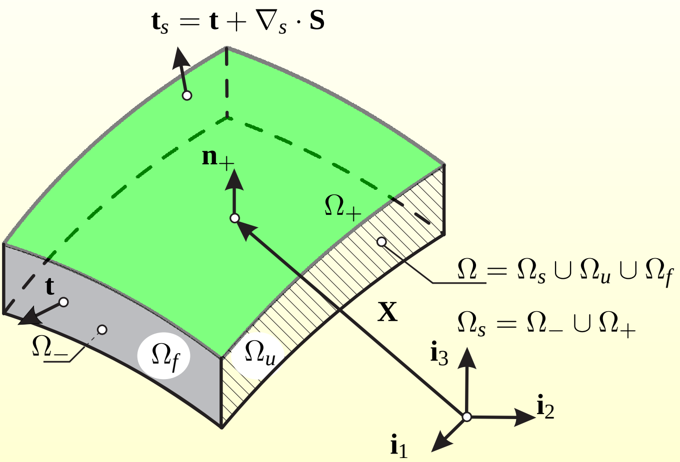
Cross-Linked Polyacrylate Nanofiber Arrays

S. Grimm et al. *Nano Lett.* , 2008.



Elastic Body with Surface Stresses

Reference configuration



Boundary-value Problem

Elastic body with surface stresses[1]:

$$\nabla_{\mathbf{x}} \cdot \mathbf{P} + \rho \mathbf{f} = \mathbf{0}\,, \quad (\mathbf{n} \cdot \mathbf{P} - \nabla_s \cdot \mathbf{S})\big|_{\Omega_s} = \mathbf{t}\,,$$
$$\mathbf{u}\big|_{\Omega_u} = \mathbf{0}\,, \quad \mathbf{n} \cdot \mathbf{P}\big|_{\Omega_f} = \mathbf{t}\,. \qquad (1)$$

Here $\mathbf{P}$ is the first Piola-Kirchhoff stress tensor, $\nabla_{\mathbf{x}}$ the 3D nabla operator, ∇_s the surface (2D) nabla operator, $\mathbf{S}$ the surface stress tensor of the first Piola-Kirchhoff type acting on the surfaces Ω_s, $\mathbf{u} = \mathbf{x} - \mathbf{X}$ the displacement vector, $\mathbf{f}$ and $\mathbf{t}$ the body force and surface loads vectors, respectively, and ρ the density.
We assume that the part of body surface Ω_u is fixed, while on Ω_f the surface stresses are absent.

[1]Gurtin and Murdoch, Arch. Rat. Mech. Analysis, 1975

Basic assumptions

Additional constitutive equation for surface

$$U = U(\mathbf{F}), \quad \mathbf{F} = \nabla_s \boldsymbol{x}_s, \quad \mathbf{S} = \frac{\partial U}{\partial \mathbf{F}},$$

or more general equations like as for example

$$U = U(\mathbf{F}, \nabla_s \mathbf{F}, \ldots).$$

Compatibility

$$\boldsymbol{x}_s \equiv \boldsymbol{x}\big|_{\Omega_s}$$

or more general like as

$$\boldsymbol{x}_s \equiv \mathcal{A}\left(\boldsymbol{x}\big|_{\Omega_s}\right)$$

Constitutive Relations

For the bulk material we use the relations

$$\mathbf{P} = \partial W / \partial \nabla_{\mathbf{x}} \mathbf{x},$$

where W is the strain energy density.
In the theory of Gurtin & Murdoch (1975) the tensor $\mathbf{S}$ is similar to the membrane stress resultants.

$$\mathbf{S} = \partial U / \partial \mathbf{F},$$

where U is the surface strain energy density.
Residual stresses: In this case we assume that W and $\mathbf{P}$ possess the properties $\mathcal{W}(\mathbf{I}) = 0, \quad \mathbf{P}(\mathbf{I}) = \mathbf{0}$, while there exist residual (initial) surface energy and surface stresses that is

$$U(\mathbf{A}) = U_0 \neq 0, \quad \mathbf{S}(\mathbf{A}) = \mathbf{S}_0 \neq \mathbf{0},$$

where $\mathbf{I}$ and $\mathbf{A} \equiv \mathbf{I} - \mathbf{N} \otimes \mathbf{N}$ are the 3D and surface unit tensors, respectively. Further we consider the influence of U_0 and $\mathbf{S}_0$ on the effective (apparent) properties of solids.

Linearized Relations

In the case of infinitesimal strains of an isotropic body we have the following constitutive equations

$$\mathbf{P} = 2\mu\boldsymbol{\varepsilon} + \lambda\mathbf{I}\operatorname{tr}\boldsymbol{\varepsilon}, \quad \mathbf{S} = \mathbf{S}_0 + 2\mu_S\boldsymbol{\epsilon} + \lambda_S\mathbf{A}\operatorname{tr}\boldsymbol{\epsilon} + \mathbf{S_0}\cdot\nabla_s\mathbf{u},$$

where

$$\boldsymbol{\varepsilon} = \frac{1}{2}\left(\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T\right), \quad \boldsymbol{\epsilon} = \frac{1}{2}\left(\nabla_S\boldsymbol{v}_s\cdot\mathbf{A} + \mathbf{A}\cdot(\nabla_S\boldsymbol{v}_s)^T\right),$$

$\mathbf{I}$ the 3D unit tensor, $\mathbf{A} \equiv \mathbf{I} - \boldsymbol{n}\otimes\boldsymbol{n}$, $\boldsymbol{v}_s = \boldsymbol{u}\Big|_{\Omega_s}$.

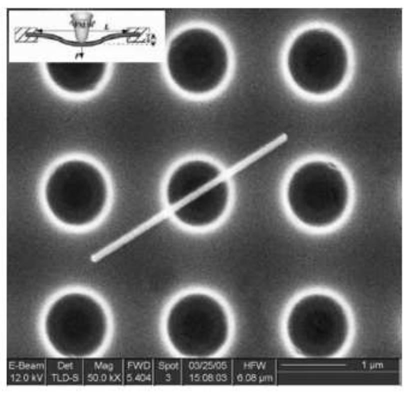
See restrictions for λ, μ and λ_S, μ_S in Altenbach et al. (2010); Javili and Steinmann (2012):

$$\mu > 0, \quad 3\lambda + 2\mu > 0; \quad \mu_S > 0, \quad \lambda_S + \mu_S > 0.$$

But $\mathbf{S}_0$ is an arbitrary second-order tensor, in general.

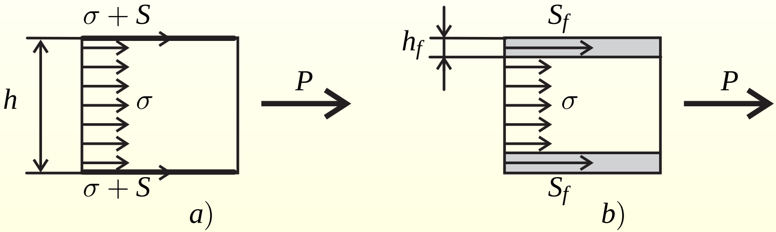
Surface Moduli Identification Problem[2]

Surface moduli $\lambda^S_\pm, \quad \mu^S_\pm$ can be determined together with the bulk moduli. A possible way is the use of the size effect, i.e. this means the using the experiments for beams with various cross-section diameters.



[2]Cuenot et al. (2004)

Comparison of three-layered plate and plate with surface stresses



For $h_\mathrm{f} \to 0$ with accuracy up to $O(h_\mathrm{f}^2)$ from comparison of the tangential and bending stiffness parameters is follows that[3]

$$\mu_S = \lim_{h_\mathrm{f}\to 0} \mu_\mathrm{f} h_\mathrm{f}, \quad \lambda_S = \lim_{h_\mathrm{f}\to 0} \lambda_\mathrm{f} \frac{1-2\nu_\mathrm{f}}{1-\nu_\mathrm{f}} h_\mathrm{f}.$$

[3]Altenbach et al. Mechanics of Solids. 2010

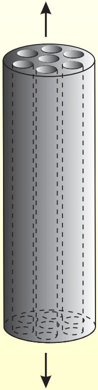
Porous rod[4]

Tension–compression, elementary formula of the strength of materials

$$E^*_\circ = E\,(1-\varphi)$$

where $\varphi = S/F$ is the porosity.

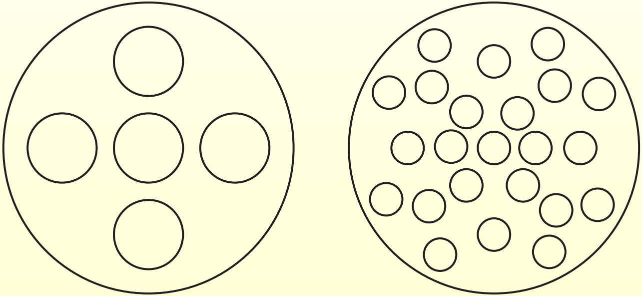
- No influence of surface effects



[4] Eremeyev and Morozov, Doklady Physics, 2010

Question: For which cross sections shown in Figure the effective Young's modulus is higher?

Two cross sections of the rod with the identical porosity (with an identical pore area S)



Effective Young's Moduli. Various Approaches

- Theory of surface stresses

$$E_S^* = E(1-\varphi) + E_S \frac{2\sqrt{S}}{\sqrt{\pi F}}\sqrt{n} = E_\circ^* + E_S \frac{2\sqrt{S}}{\sqrt{\pi F}}\sqrt{n}.$$

- Surface layer (Mechanics of composites)

$$E_f^* = E\left(1 - \frac{S+S_\delta}{F}\right) + E_f\frac{S_\delta}{F} = E_\circ^* + (E_f - E)\frac{S_\delta}{F},$$

where $S_\delta(n) = \pi n[(r+\delta)^2 - r^2]$ is the area of the surface layers and, finally,

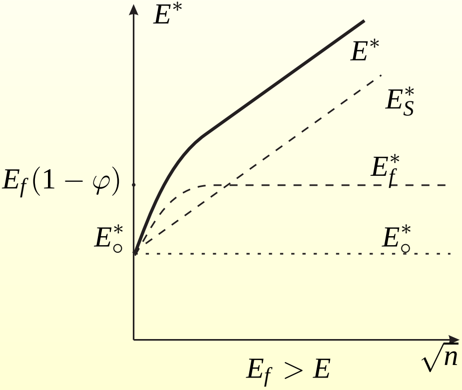
$$E_f^* = E_\circ^* + (E_f - E)\frac{2\delta\sqrt{\pi S}}{F}\sqrt{n}.$$

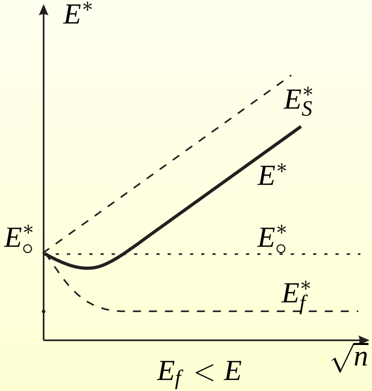
- Complex formula

$$E^* = E_\circ^* + E_S\frac{2\sqrt{S}}{\sqrt{\pi F}}\sqrt{n} + (E_f - E)\frac{S_\delta(n)}{F}.$$

Effective Young's Moduli. Various Approaches

Dependencies of E^*, $E^*_\circ$, E^*_S and E^*_f on $\sqrt{n}$ for $E_f > E$ (on the left) and for $E_f < E$ (on the right)

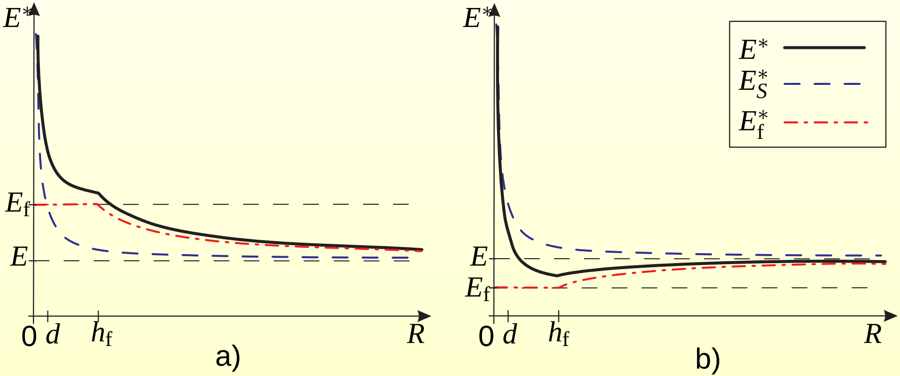




Effective Young's Moduli. Various Approaches. II

E^* depends on the values of h_f, R, E_S, and E. Here $d = 2E_S/E$ is the characteristic length parameter introduced in Duan et al. (2008), Wang et al. (2006).

Effective Young modulus E^* as the function of radius R: a) $E_f > E$, b) $E_f < E$.



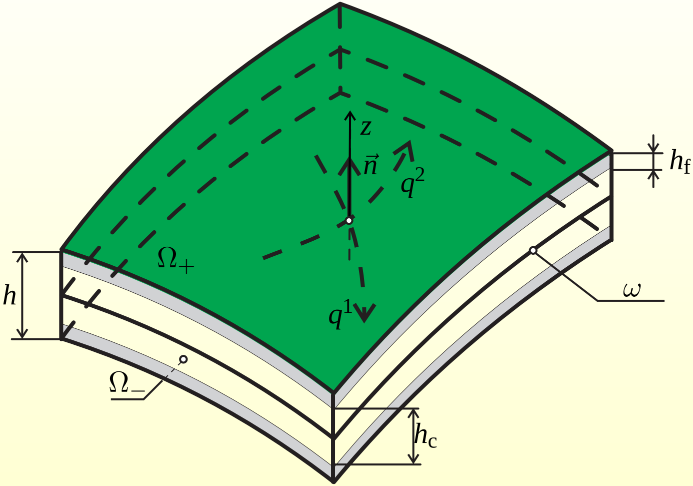
Two-dimensional Theories of Nanosized Plates and Shells

The theory of elasticity with surface stresses is applied to the modifications of the two-dimensional theories of nanosized plates and shells:

- Miller & Shenoy (2000);
- Dahmen, Lehwald & Ibach (2000);
- Lu, He, Lee & Lu (2006);
- Huang (2008);
- Lu, Lim & Chen (2009);
- Eremeyev, Altenbach & Morozov (2009a,b, 2010, 2012);
- Altenbach & Eremeyev (2010, 2011).

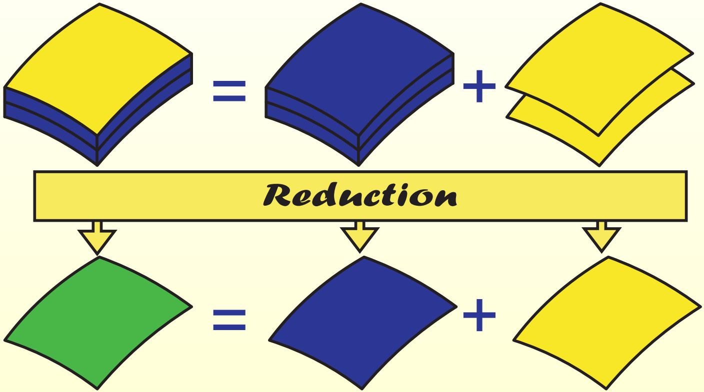
Various theories of plates are formulated, i.e. Kirchhoff–Love, Reissner & Mindlin, von Kàrmàn, and the 6-parameters theory by Libai & Simmonds among others

Shell-like body



3D to 2D Reduction

$$\mathbf{T}^* = \mathbf{T} + \mathbf{T}_S\,, \qquad \mathbf{M}^* = \mathbf{M} + \mathbf{M}_S\,,$$



$\mathbf{T}^*, \mathbf{M}^*$ $\mathbf{T}, \mathbf{M}$ $\mathbf{T}_S, \mathbf{M}_S$

2D equilibrium equations

$$\nabla_S \cdot \mathbf{T} + \boldsymbol{q} = \mathbf{0}, \quad \nabla_S \cdot \mathbf{M} + \mathbf{T}_\times + \boldsymbol{m} = \mathbf{0},$$

where $\mathbf{T}$ and $\mathbf{M}$ are the stress resultant and stress couple tensors, respectively, $\boldsymbol{q}$ and $\boldsymbol{m}$ are the external surface loads and moments, and $\mathbf{T}_\times$ denotes the vectorial invariant of the second-order tensor $\mathbf{T}$.

$$\mathbf{T} = \langle (\mathbf{A} - z\mathbf{B})^{-1} \cdot \boldsymbol{\sigma} \rangle + \mathbf{S}_+ + \mathbf{S}_-, \quad \langle (\ldots) \rangle = \int_{-h/2}^{h/2} (\ldots) G \, dz,$$

$$\mathbf{M} = -\langle (\mathbf{A} - z\mathbf{B})^{-1} \cdot z\boldsymbol{\sigma} \times \boldsymbol{n} \rangle - \frac{h}{2} (\mathbf{S}_+ - \mathbf{S}_-) \times \boldsymbol{n},$$

$$\boldsymbol{q} = G_+ \boldsymbol{\varphi}_+ - G_- \boldsymbol{\varphi}_-, \quad \boldsymbol{m} = \frac{h}{2} G_+ \boldsymbol{n} \times \boldsymbol{\varphi}_+ + \frac{h}{2} G_- \boldsymbol{n} \times \boldsymbol{\varphi}_-,$$

$$G = G(z) \equiv \det(\mathbf{A} - z\mathbf{B}), \quad G_\pm = G(\pm h/2), \quad \mathbf{B} = -\nabla_S \boldsymbol{n}.$$

If $h\|\mathbf{B}\| \ll 1$ then

$$\mathbf{T} = \langle \mathbf{A} \cdot \boldsymbol{\sigma} \rangle + \mathbf{S}_+ + \mathbf{S}_-, \quad \mathbf{M} = -\langle \mathbf{A} \cdot z\boldsymbol{\sigma} \times \boldsymbol{n} \rangle - \frac{h}{2} (\mathbf{S}_+ - \mathbf{S}_-) \times \boldsymbol{n}.$$

2D constitutive equations

Kinematic assumptions

$$\boldsymbol{u}(q^1, q^2, z) = \boldsymbol{w}(q^1, q^2) - z\boldsymbol{\vartheta}(q^1, q^2), \quad \boldsymbol{n}\cdot\boldsymbol{\vartheta} = 0.$$

Constitutive relations

$$\mathbf{T} = C_1\mathbf{E} + C_2\mathbf{A}\,\mathrm{tr}\,\mathbf{E} + \Gamma\boldsymbol{\gamma}\otimes\boldsymbol{n}, \quad \mathbf{M} = -\left[D_1\mathbf{K} + D_2\mathbf{A}\,\mathrm{tr}\,\mathbf{K}\right]\times\boldsymbol{n},$$

where $\mathbf{E}$, $\mathbf{K}$, and γ are the surface strain measures given by

$$\mathbf{E} = \frac{1}{2}\left(\nabla_S\boldsymbol{w}\cdot\mathbf{A} + \mathbf{A}\cdot(\nabla_S\boldsymbol{w})^T\right), \quad \mathbf{K} = \frac{1}{2}\left(\nabla_S\boldsymbol{\vartheta}\cdot\mathbf{A} + \mathbf{A}\cdot(\nabla_S\boldsymbol{\vartheta})^T\right),$$

$$\boldsymbol{\gamma} = \nabla_S(\boldsymbol{w}\cdot\boldsymbol{n}) - \boldsymbol{\vartheta},$$

C_1, C_2 are the tangential stiffness parameters, D_1 and D_2 are the bending stiffness parameters, and Γ is the transverse shear stiffness.

Stiffness parameters

$$C_1 = 2C_{22} + 4\mu_S, \quad C_2 = C_{11} - C_{22} + 2\lambda_S,$$

$$D_1 = 2D_{22} + h^2\mu_S, \quad D_2 = D_{33} - D_{22} + \frac{h^2}{2}\lambda_S, \quad \Gamma = \ell^2 D_{22},$$

$$C_{11} = \frac{1}{2}\left(\frac{2E_\mathrm{f}h_\mathrm{f}}{1-\nu_\mathrm{f}} + \frac{Eh_\mathrm{c}}{1-\nu}\right), \quad C_{22} = \frac{1}{2}\left(\frac{2E_\mathrm{f}h_\mathrm{f}}{1+\nu_\mathrm{f}} + \frac{Eh_\mathrm{c}}{1+\nu}\right),$$

$$D_{22} = \frac{1}{24}\left[\frac{E_\mathrm{f}(h^3 - h_\mathrm{c}^3)}{1+\nu_\mathrm{f}} + \frac{Eh_\mathrm{c}^3}{1+\nu}\right], \quad D_{33} = \frac{1}{24}\left[\frac{E_\mathrm{f}(h^3 - h_\mathrm{c}^3)}{1-\nu_\mathrm{f}} + \frac{Eh_\mathrm{c}^3}{1-\nu}\right],$$

where ℓ is the minimal positive root of the following equation

$$\mu_0 \cos\ell\frac{h_\mathrm{f}}{2}\cos\ell\frac{h_\mathrm{c}}{2} - \sin\ell\frac{h_\mathrm{f}}{2}\sin\ell\frac{h_\mathrm{c}}{2} = 0, \quad \mu_0 = \mu_\mathrm{c}/\mu_\mathrm{f},$$

μ and μ_f are the shear moduli of the shell core and faces, respectively.

Tangential and bending stiffness

The effective tangential and bending stiffness take the form

$$C^* \equiv C_1 + C_2 = \frac{2E_\mathrm{f} h_\mathrm{f}}{1-\nu_\mathrm{f}^2} + \frac{E h_\mathrm{c}}{1-\nu^2} + 4\mu_S + 2\lambda_S,$$

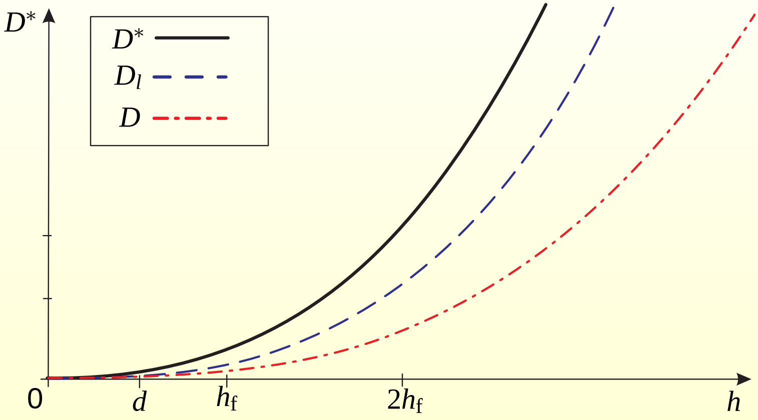
$$D^* \equiv D_1 + D_2 = \frac{1}{12}\left[\frac{E_\mathrm{f}(h^3 - h_\mathrm{c}^3)}{1-\nu_\mathrm{f}^2} + \frac{E h_\mathrm{c}^3}{1-\nu^2}\right] + \frac{h^2}{2}(2\mu_S + \lambda_S).$$

The classical bending stiffness D and the bending stiffness of the three-layered plate D_l are given by

$$D = \frac{Eh^3}{12(1-\nu^2)}, \quad D_l = \frac{1}{12}\left[\frac{E_\mathrm{f}(h^3 - h_\mathrm{c}^3)}{1-\nu_\mathrm{f}^2} + \frac{E h_\mathrm{c}^3}{1-\nu^2}\right],$$

and is assumed to be $E_\mathrm{f} > E$.

Bending stiffness

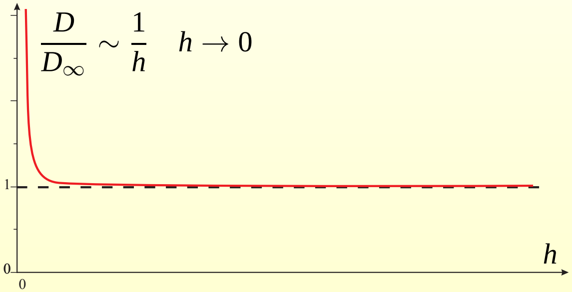


Bending Stiffness

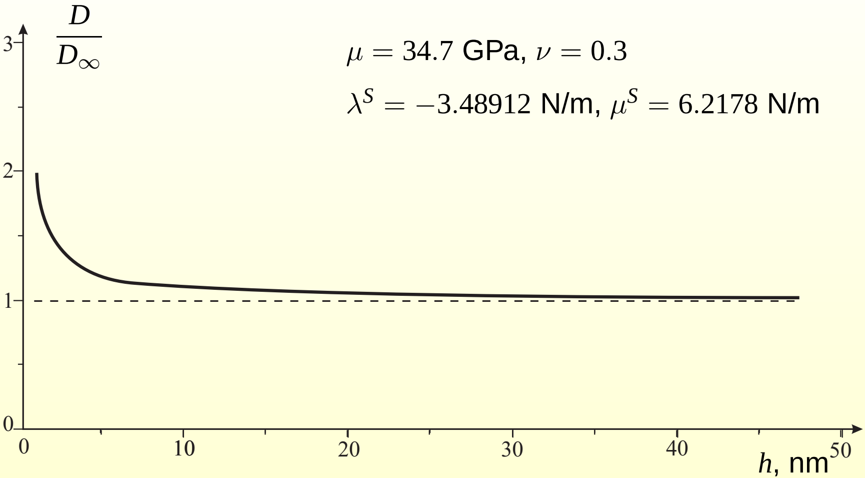
$$D \equiv D_1 + D_2 = D_\infty + D_{surface},$$
$$D_\infty = \frac{Eh^3}{12(1-\nu^2)}, \quad D_{surface} = h^2(\mu^S + \lambda^S/2) \qquad (2)$$

From the positivity of the surface energy density it follows

$$\mu_S > 0, \quad \mu_s + \lambda_S > 0 \quad \Longrightarrow \quad D_{surface} > 0$$



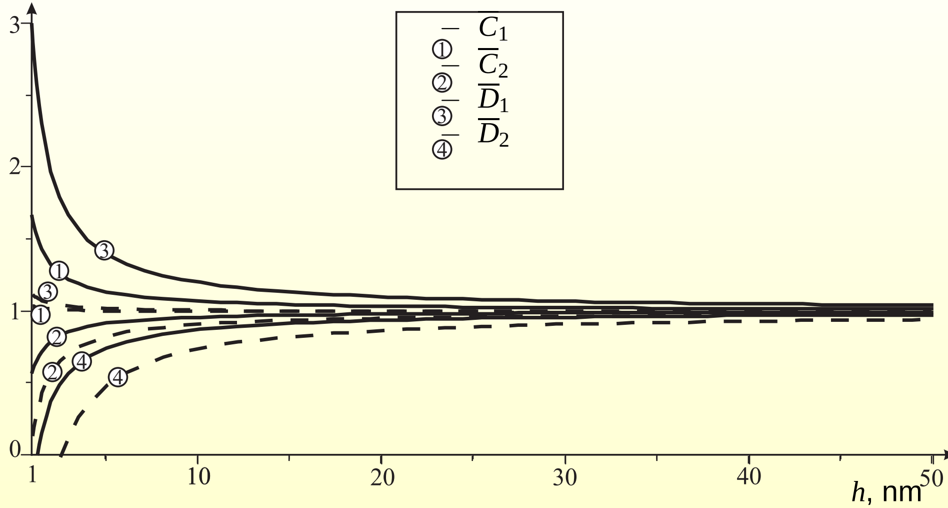
Bending Stiffness of a Plate made of Al



$\mu = 34.7$ GPa, $\nu = 0.3$

$\lambda^S = -3.48912$ N/m, $\mu^S = 6.2178$ N/m

Stiffness Parameters $\overline{C}_1 = C_1/C(1-\nu)$, $\overline{C}_2 = C_2/C\nu$, $\overline{D}_1 = D_1/D(1-\nu)$, $\overline{D}_2 = D_2/D\nu$



Homogenization+Homogenization: two steps of Homogenization

On effective surface properties

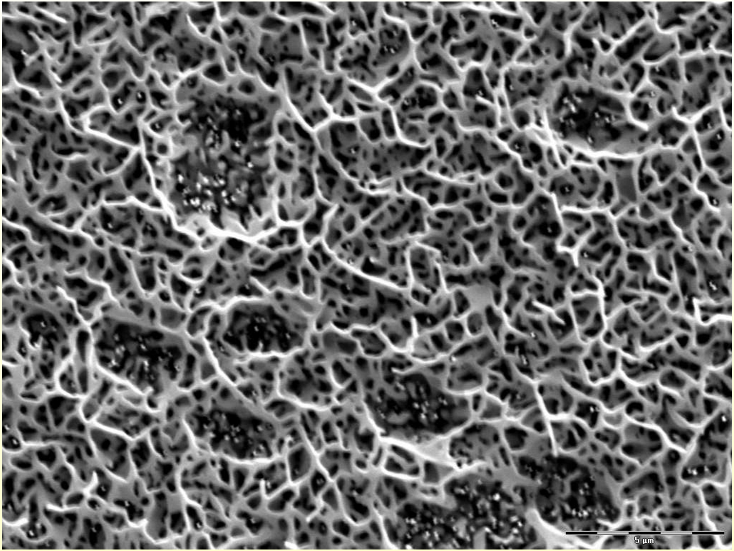
- Find 2D effective (apparent) material properties.

On effective bulk properties

- Using these 2D properties find 3D effective material properties.

“Foam-like” surface

ZnO nanofoam grown on the glass substrate.



Effective properties for “foam-like” surface

On effective bulk properties

Following Gibson and Ashby (1997)

$$\frac{E_{\rm f}}{E_{\rm b}} \sim \alpha^m, \qquad \frac{G_{\rm f}}{G_{\rm sb}} \sim \alpha^m,$$

where α is the porosity, $m \approx 2$, $\nu_f \approx 0.3$

On effective bulk properties at the nanoscale

Following Wang et al. (2006) we assume the scaling law

$$E_{\rm n} = E_{\rm s}\left(1 + \chi\frac{l_{\rm in}}{R}\right),$$

where $l_{\rm in}$ an intrinsic length scale related to the surface properties, R is the specimen size, and χ a nondimensional constant.

Effective properties for “foam-like” surface. II

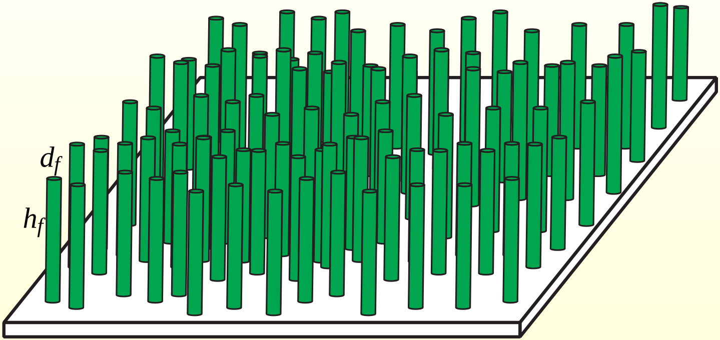
Scaling law

We modify the dependence of the elastic moduli of a nanofoam on the porosity by the law

$$\frac{E_{\rm np}}{E_{\rm b}} \sim \left(1+\chi\frac{l_{\rm in}}{R}\right)\alpha^m, \qquad \frac{G_{\rm np}}{G_{\rm b}} \sim \left(1+\chi\frac{l_{\rm in}}{R}\right)\alpha^m,$$

where $E_{\rm np}$ and $G_{\rm np}$ are the Young’s and shear moduli of a nanofoam, respectively. $l_{\rm in}$ is related to the surface effects and is typically in the order of $0.01-0.1$nm, see Duan et al. (2008), Wang et al. (2006).

Array of fibers



d_f and h_f are the diameter and hight, N denotes the number of fibers per unit area.

"Averaged" properties of layer of fibers

A transversely isotropic material

The longitudinal Young's modulus

$$E_{\perp} = NC_f.$$

The in-plane shear modulus

$$G_f = \frac{12N}{h_f^2} D_f.$$

Other three elastic moduli are determined by interaction forces between fibers, that is adhesion-type forces.

$C_f = \frac{\pi d_f^2}{4} E_f$ and D_f are the tangential and bending stiffness of the fiber.

Conclusions and Future Steps

- We discussed the effective surface properties taking into account the surface stresses.
- We found the few expressions for effective stiffness parameters of plates and shells.
- In particular, the bending stiffness is bigger for the shells with surface stresses than for shells without surface elasticity.
- The surface residual stresses influence the effective stiffness parameters making the body more or less stiffer.
- Inner structure of the “surface” leads to changes in effective properties of materials at the nano- and microscales.

References

- Altenbach, H., Eremeyev, V.A., Lebedev, L.P.: On the existence of solution in the linear elasticity with surface stresses. *ZAMM* **90**(7), 535–536 (2010)
- Altenbach, H., Eremeyev, V.A., Lebedev, L.P.: On the spectrum and stiffness of an elastic body with surface stresses. *ZAMM* **91**(9), 699–710 (2011)
- Altenbach, H., Eremeyev, V.A.: On the shell theory on the nanoscale with surface stresses. *Int. J. Engng Sci.* **49**(12), 1294–1301 (2011)
- Altenbach, H., Eremeyev, V.A., Morozov, N.F.: On equations of the linear theory of shells with surface stresses taken into account. *Mechanics of Solids* **45**(3), 331–342 (2010)
- Altenbach, H., Eremeyev, V.A., Morozov, N.F.: Surface viscoelasticity and effective properties of thin-walled structures at the nanoscale. *Int. J. Engng Sci.* **59**, 83–89 (2012)
- Altenbach, H., Morozov, N.F. (eds.): *Surface Effects in Solid Mechanics – Models, Simulations, and Applications*. Springer, Berlin (2013)

Thank you for your attention!!!

Further questions:

eremeyev.victor@gmail.com