PhD course: Geometric Analysis
Lecturer: prof. A. Greco

STUDENTS' SEMINARS

Each seminar should be self-contained and have the following structure:

1. An introduction that settles the body of the talk in a context and motivates the result;

2. The body of the seminar (see below);

3. Conclusions, indicating for instance potential applications or further developments of the subject.

Duration: 30 minutes including questions.

Students must communicate in advance the topic of their seminar to the lecturer for approval. The topic may be chosen from the list below. Students may also propose an alternative topic of their own choice.

Seminar 1

State and prove: Green's first identity, Green's second identity, Green's representation formula.
Then derive either Gauss' theorem of the mean from Green's representation formula, or the analyticity of harmonic functions.

Reference: Sobolev, Partial differential equations of mathematical physics.
Location: MECCANICA M 24 0019
See, in particular: Theorem 1 on p. 156 (theorem of the mean) or pp. 155-156 (analyticity)
*** Students may refer to alternative references of their own choice ***

Seminar 2

State and prove the converse of Gauss' theorem of the mean, sometimes called Koebe's theorem.

Reference: Kellogg, Foundations of potential theory.
Location: ANALISI 515.9 KEL
See, in particular: Chapter VIII, sect. 7
*** Students may refer to alternative references of their own choice ***

Seminar 3

Define the weakly harmonic functions and prove their harmonicity (Weyl's lemma).

Reference: Pucci, Istituzioni di analisi superiore (Italian).
Location: DID 510 PUC
*** Students may refer to alternative references of their own choice ***

Seminar 4

Prove the locally uniform pre-compactness of bounded sequences of harmonic functions.

Reference: Gilbarg & Trudinger, Elliptic partial differential equations of second order.
Location: ANALISI AMS 35G 0003
See, in particular, Theorem 2.11
*** Students may refer to alternative references of their own choice ***

(see also p. 2)

Seminar 5

Prove Weinberger's version of Serrin's theorem.

Reference: Weinberger, Remark on the preceding paper of Serrin.
Arch. Rational Mech. Anal. 43 (1971), 319–320.

Seminar 6

Prove the Hopf boundary point lemma for the fractional Laplacian.

Reference: Greco & Servadei, Hopf's lemma and constrained radial symmetry for the fractional Laplacian.
Location:
https://www.intlpress.com/site/pub/files/_fulltext/journals/mrl/2016/0023/0003/MRL-2016-0023-0003-a014.pdf
See, in particular, Lemma 3.1 on p. 873
Math. Res. Lett. 23(3) (2016), 863–885.
*** Students may refer to alternative references of their own choice ***

Seminar 7

Prove Serrin's corner lemma, also called Lemma (S) or edge-point lemma.

Reference: Serrin, A symmetry problem in potential theory.
Arch. Rational Mech. Anal. 43 (1971), 304–318.
See Lemma 1 on p. 308
Location: http://web.math.unifi.it/users/magnanin/Dott/BibliografiaCorso/Serrin71.pdf
*** Students may refer to alternative references of their own choice ***

Seminar 8

Replace $\partial u/\partial n = c$ in Serrin's theorem with $\partial u/\partial n = c|x|$ and prove radial symmetry about the origin.

Reference: Greco, Symmetry around the origin for some overdetermined problems.
Adv. Math. Sci. Appl. 13 (2003), 387–399.
See Theorem 1 on p. 387

Seminar 9

Extend Serrin's symmetry result to domains with cavities

Reference: Alessandrini, A symmetry theorem for condensers.
Math. Methods Appl. Sci. 15 (1992), 315–320.
It is enough to consider $a\equiv 1$ (Laplace operator) and $N=1$ (one cavity)

Seminar 10

Prove that the logarithm of the first eigenfunction of the Dirichlet-Laplacian is strictly concave near the boundary of a strictly convex domain

Reference: Greco, On the curvature of functions blowing up on the boundary.
See Theorem 2.1
Location: https://web.unica.it/unica/protected/438614/0/def/ref/MAT430749/
*** Students may refer to alternative references of their own choice ***
