Theoretical and numerical analysis of differential equations and their application

ERASMUS Blended Intensive Program, 21-25 July 2025. Cagliari, Italy

Courses

Preface

This document contains the abstracts for the 5-day course, ***Theoretical and Numerical Analysis of Differential Equations and Their Applications***, held at the University of Cagliari. The primary objective of the course is to provide a comprehensive overview of the theoretical foundations and numerical methods for differential equations, with a distinct focus on their application to problems in science and engineering.

The event is a collaborative initiative coordinated by the Department of Mathematics and Computer Science at the University of Cagliari, in partnership with the University of Cádiz and Hacettepe University. The curriculum is structured to accommodate both undergraduate and graduate students, integrating rigorous theoretical lectures with applied problem-solving sessions and worked examples.

The academic program is composed of three distinct yet complementary courses:

- General Introduction to Ordinary Differential Equations. Applications to Real-World Models
- Some Types of Linear PDEs and Systems. Numerical Methods for PDEs
- Mathematical Modelling and Applications of PDEs and Systems

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Course 1

General Introduction to Ordinary Differential Equations and Applications to Real-World Models

E. Öztürk (Hacettepe University)
F. G. Düzgün (University of Cagliari)

Many physical laws and relations can be expressed mathematically in the form of differential Equations. Modeling is a crucial general process in engineering, physics, computer science, biology, medicine, environmental science, chemistry, economics, and other fields. In this course, we introduce some types of Linear Ordinary Differential Equations of first order and Linear Systems of Ordinary Differential Equations, by their solving methods and showing Existence and Uniqueness Theorems. After that, we will study some mathematical model examples as their applications.

References

- E. Kreyszig, H. Kreyszig, and E. Norminton. *Advanced Engineering Mathematics*. 10th ed. John Wiley & Sons, Inc., 2011.
- L. J. S. Allen. *An Introduction to Mathematical Biology*. Pearson/Prentice Hall, 2007.
- S. L. Ross. *Differential Equations*. John Wiley & Sons, Inc., 2004.
- L. G. Petrovski. *Ordinary Differential Equations*. Prentice-Hall, 1966.

Course 2

Some Types of Linear PDEs and Systems and Numerical Methods for PDEs

J. R. Rodríguez Galván (University of Cádiz)

This course provides an introduction to numerical methods for solving boundary value problems involving partial differential equations (PDEs). We begin with a review of the main types of PDEs and explore techniques for numerically approximating their solutions, with a particular focus on the Finite Element Method (FEM). The course presents the necessary functional analysis framework and applies it through practical exercises that demonstrate the effectiveness of FEM in simulating processes in science and engineering.

References

- G. Allaire. *Numerical Analysis And Optimization: An introduction to mathematical modelling and numerical simulation.* Oxford University Press, 2007.
- A. Ern, J.-L. Guermond. *Theory and Practice of Finite Elements.* Springer-Verlag, 2004.

Course 3

Mathematical Modelling and Applications of PDEs and Systems

R. Díaz Fuentes

Partial Differential Equations (PDEs) are a fundamental mathematical tool for describing phenomena that vary in both space and time, with applications spanning physics, biology, and engineering. This course offers a concise introduction to modelling with PDEs, beginning with their essential classification. We will explore two foundational classes: parabolic equations, through the study of the Heat Equation to model diffusion, and elliptic equations, via the Laplace and Poisson equations for steady-state systems. The course culminates in synthesizing these concepts to construct and analyze the Keller-Segel model for chemotaxis, revealing how the interplay between parabolic diffusion and directed movement can lead to collective cell behavior.

References

- L.-C. Evans. *Partial Differential Equations.* 2nd ed. American Mathematical Society, 2010.
- L. Edelstein-Keshet. *Mathematical Models in Biology.* SIAM, 2005.
- D. Horstmann, M. Winkler. Boundedness vs. blow-up in a chemotaxis system. *Journal of Differential Equations*, 215(1), 52-107, 2005.